Graphene plasmonics

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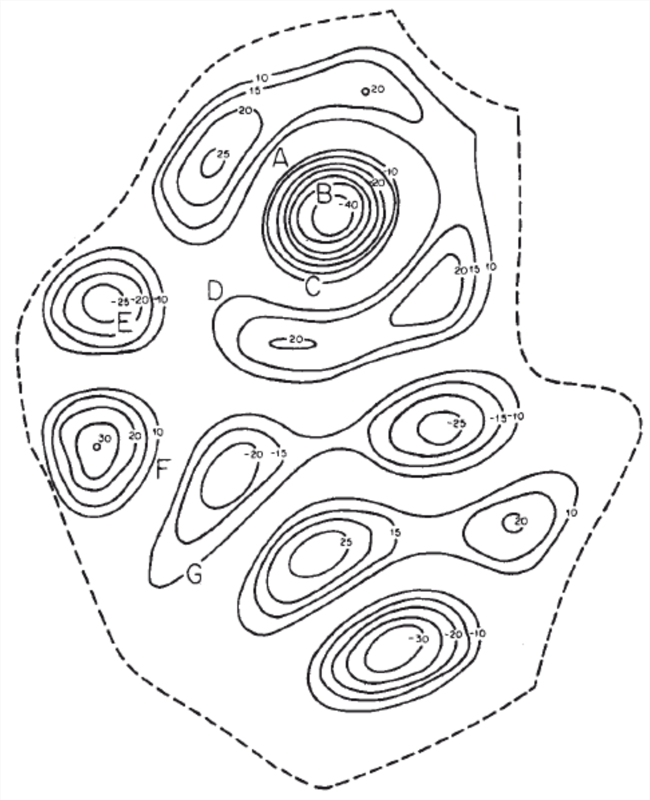
Light — Optoelectronics Research Centre

Introduction to plasmons

Surface plasmons are surface waves …

Propagating surface waves

Propagating surface waves

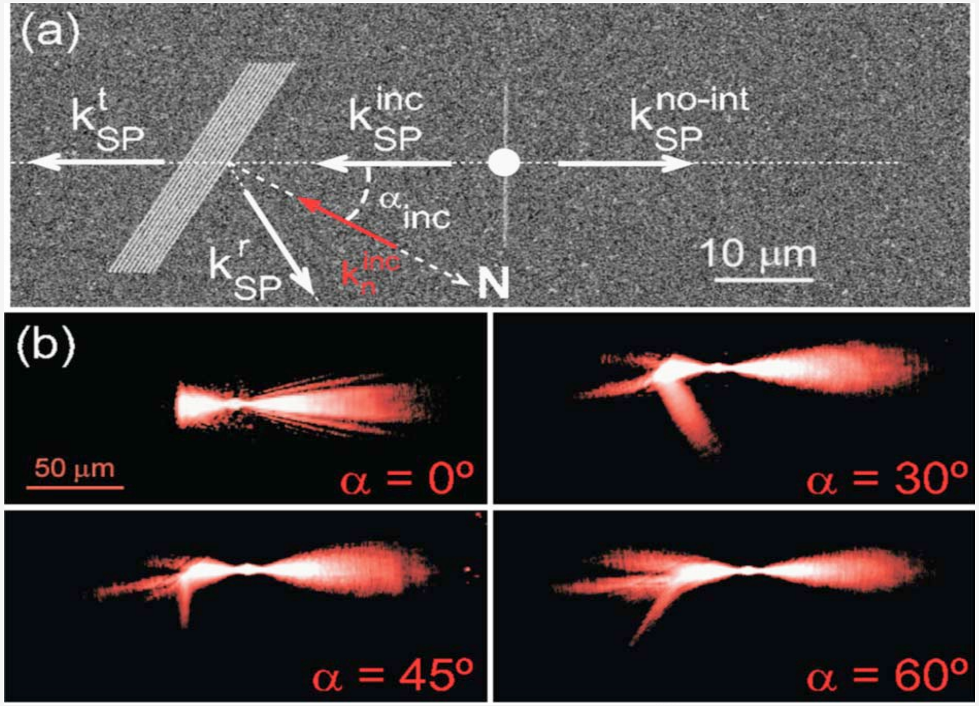


Flores *et al.*, Nature (1987)

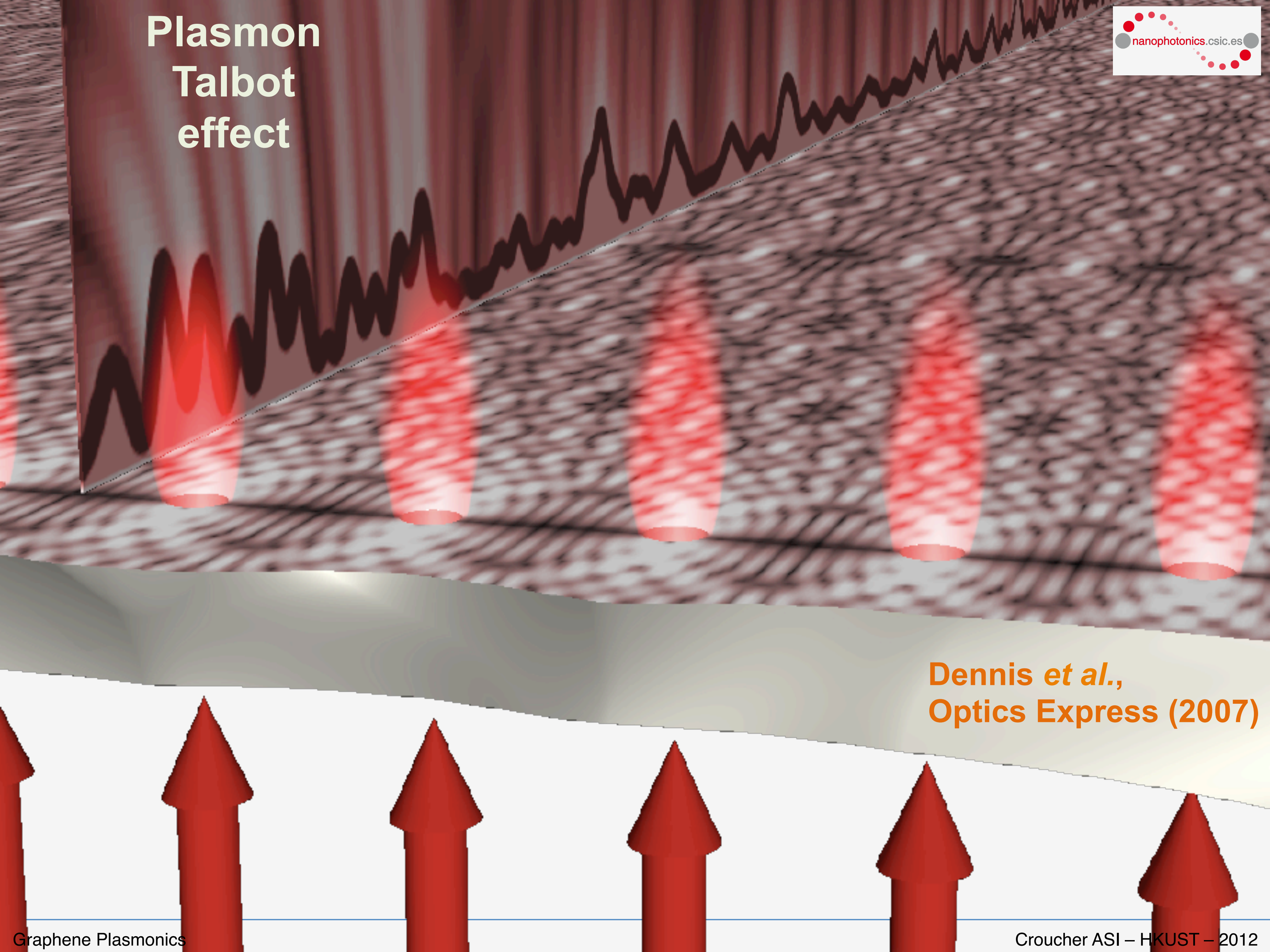
Level of damage in the 1985 earthquake in Mexico city

Surface plasmons are surface waves
involving collective electron motion
and propagating on metal surfaces …

Plasmon Bragg mirrors

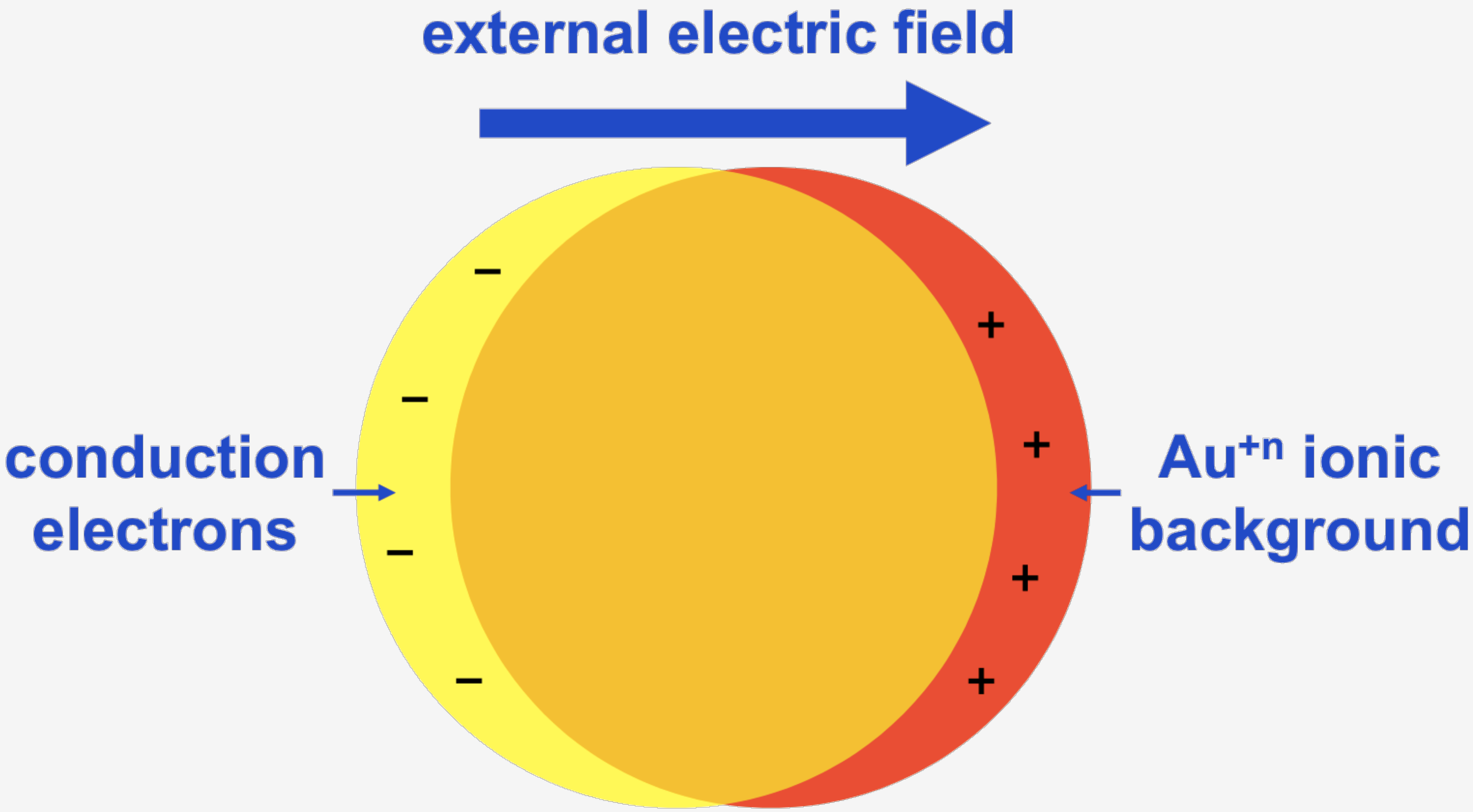


González, …, Dereux, Quidant, Krenn, Opt. Lett. (2007)


Plasmon
Talbot
effect
nanophotonics.csic.es
Dennis et al.,
Optics Express (2007)

Surface plasmons are surface waves
involving collective electron motion
and propagating on metal surfaces
or localized in metal (nano)structures
(e.g., nanoparticles), where they couple
efficiently to light …

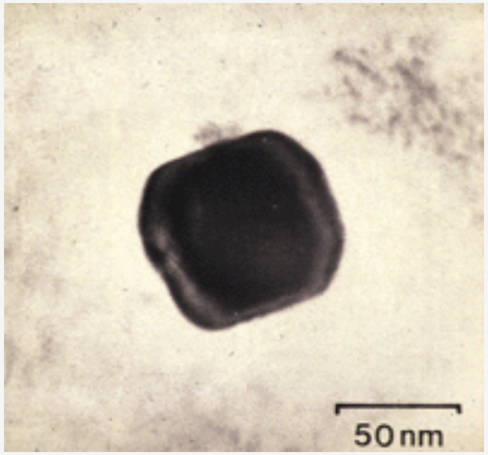
Plasmons in metallic nanoparticles



Plasmons in metallic nanoparticles

Romans played empirically with nanoparticle plasmons: the Licurgo cup dating from the IV century

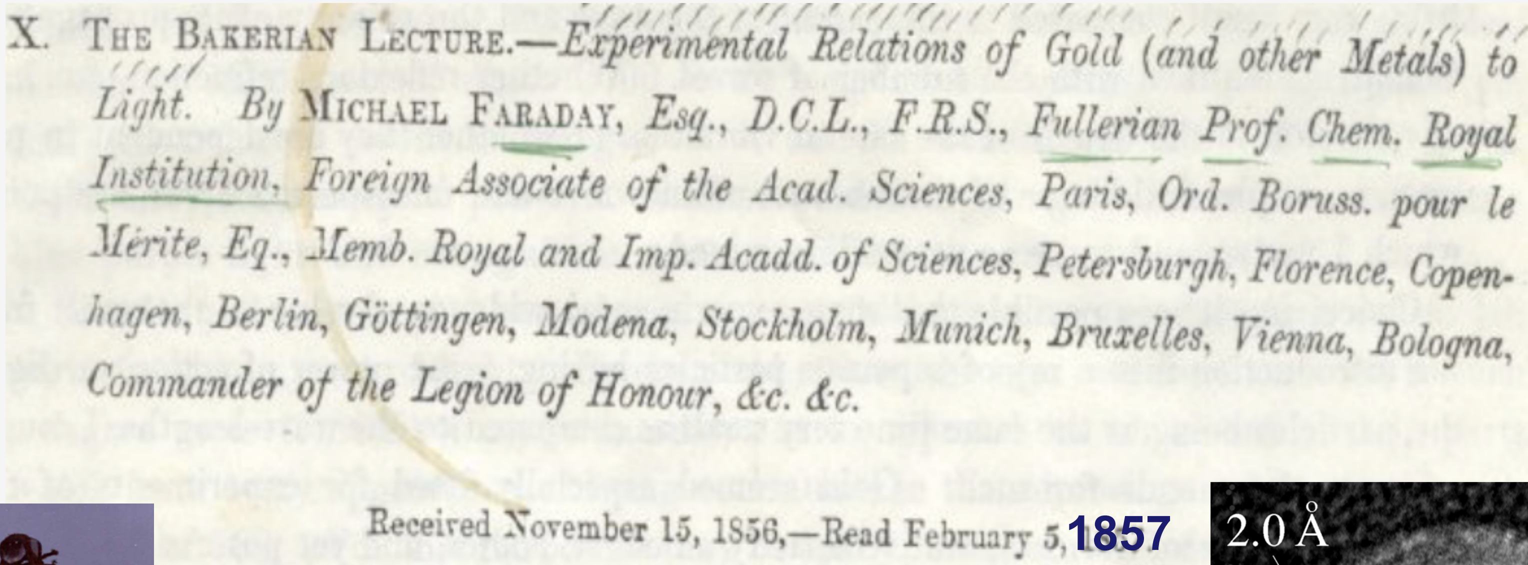
In reflection



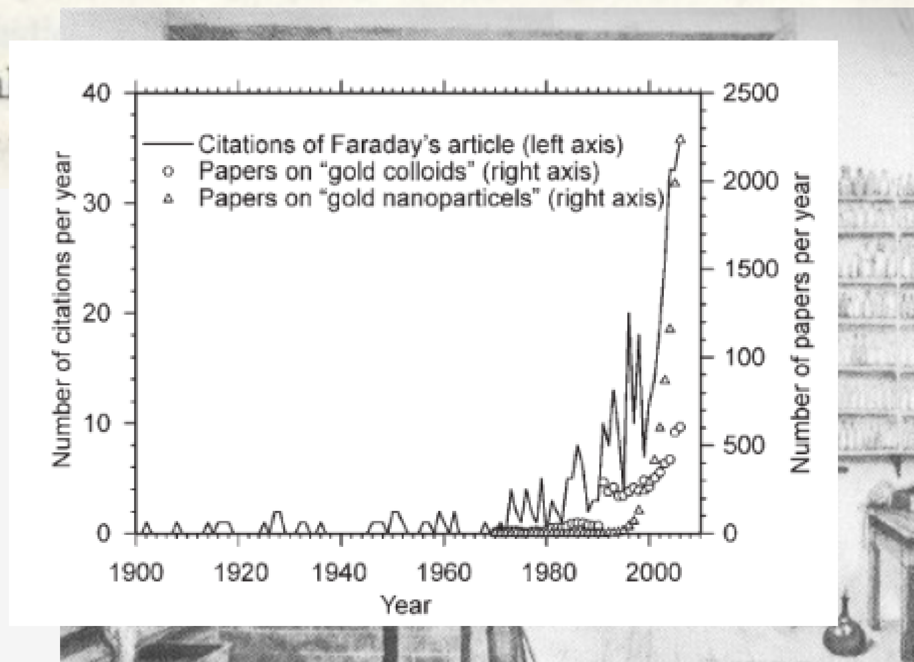
In transmission

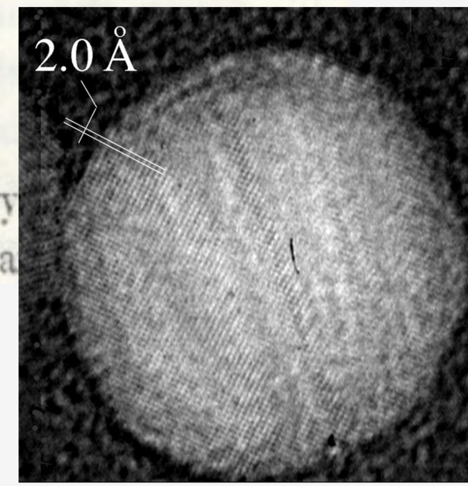
An electron microscope image shows 70-nm Au-Ag nanoparticles inside the glass

Plasmons in metallic nanoparticles


X. THE BAKERIAN LECTURE.—*Experimental Relations of Gold (and other Metals) to Light.* By MICHAEL FARADAY, *Esq., D.C.L., F.R.S., Fullerian Prof. Chem. Royal Institution, Foreign Associate of the Acad. Sciences, Paris, Ord. Boruss. pour le Mérite, Eq., Memb. Royal and Imp. Acadd. of Sciences, Petersburgh, Florence, Copenhagen, Berlin, Göttingen, Modena, Stockholm, Munich, Bruxelles, Vienna, Bologna, Commander of the Legion of Honour, &c. &c.*

Received November 15, 1856,—Read February 5, **1857**





Plasmons in metallic nanoparticles

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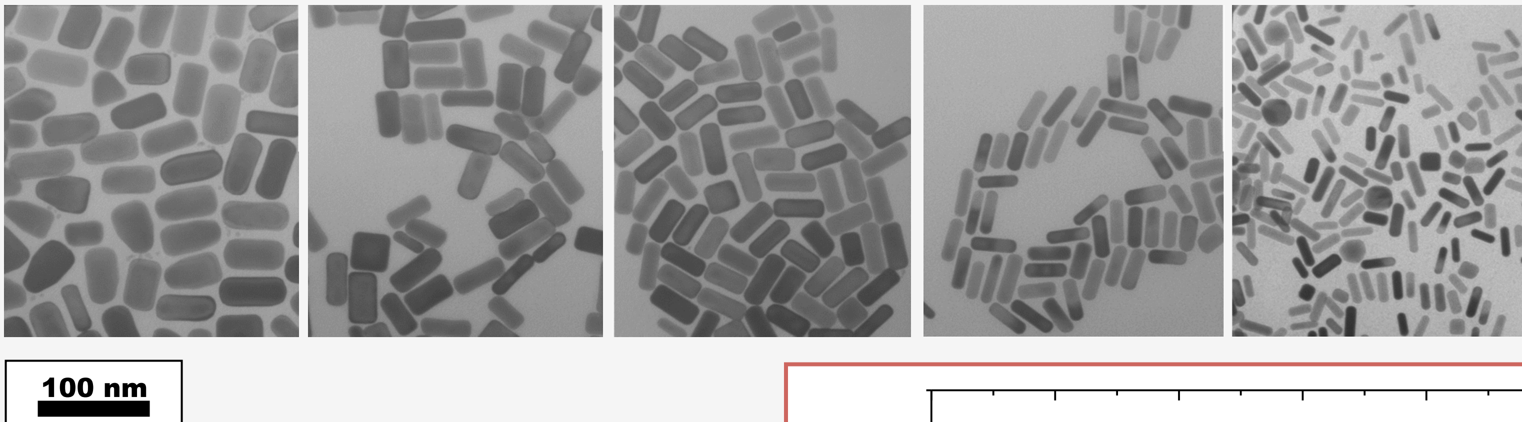
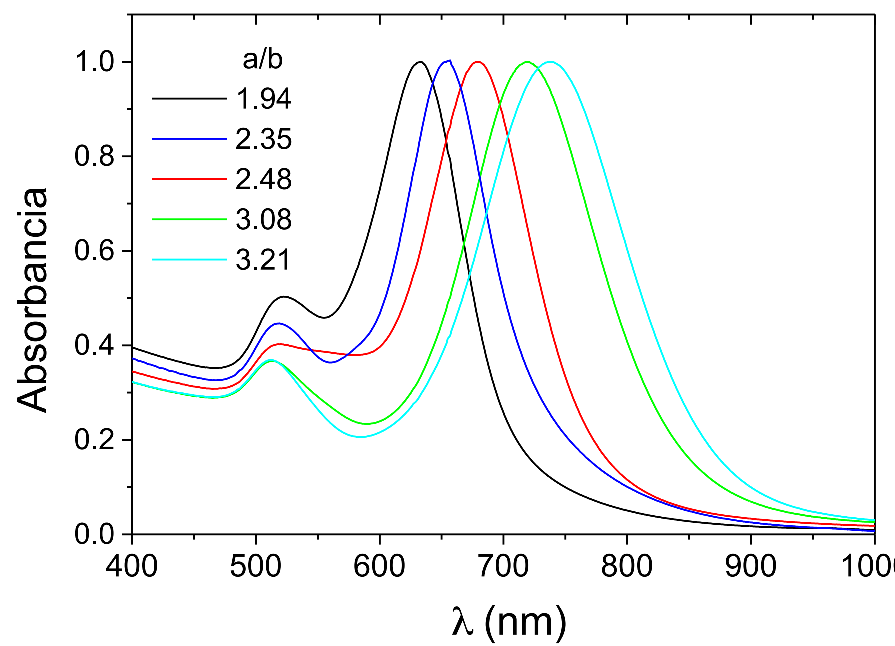
The colors of gold nanorods

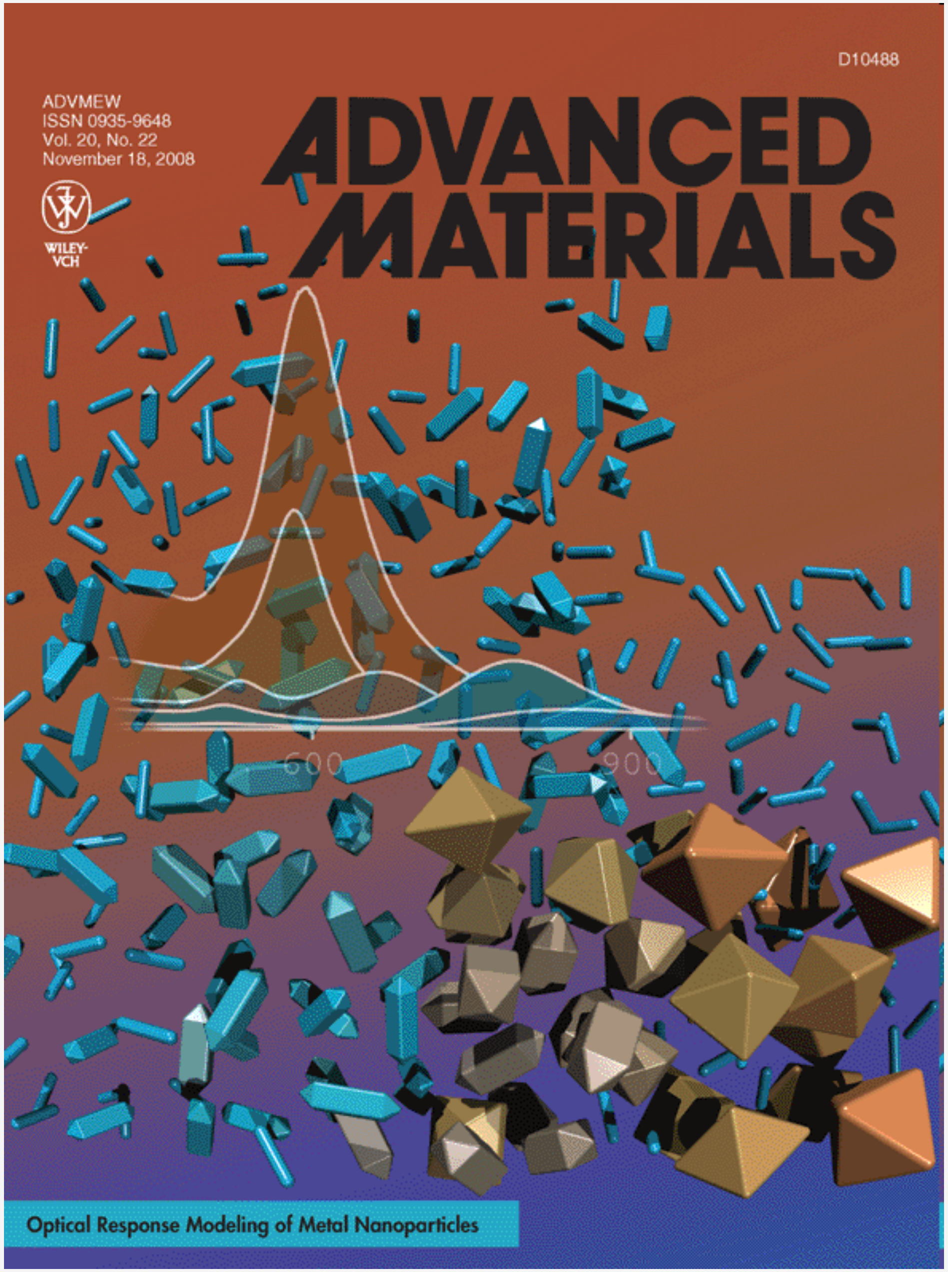
AR 1,94 2,35 2,48 3,08
3,21





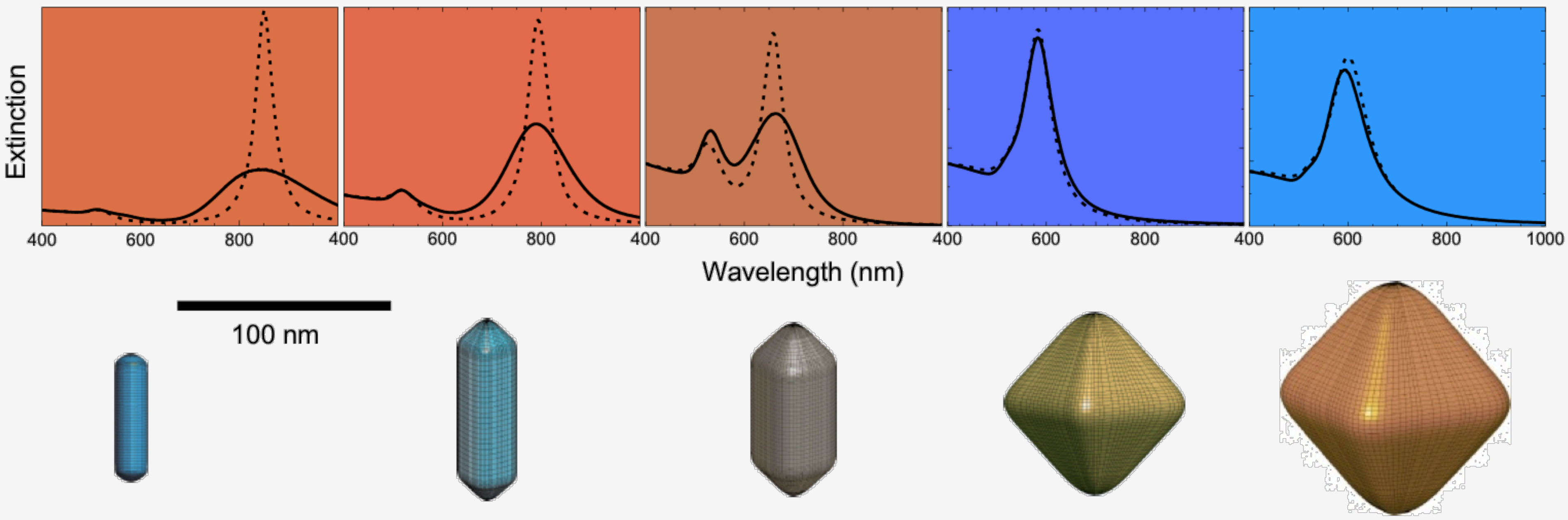
Pérez-Juste *et al*., Appl. Surf. Sci. (2004)

Plasmons in metallic nanoparticles



Plasmons in metallic nanoparticles

Artificial colors through tailored plasmons in nanoparticles



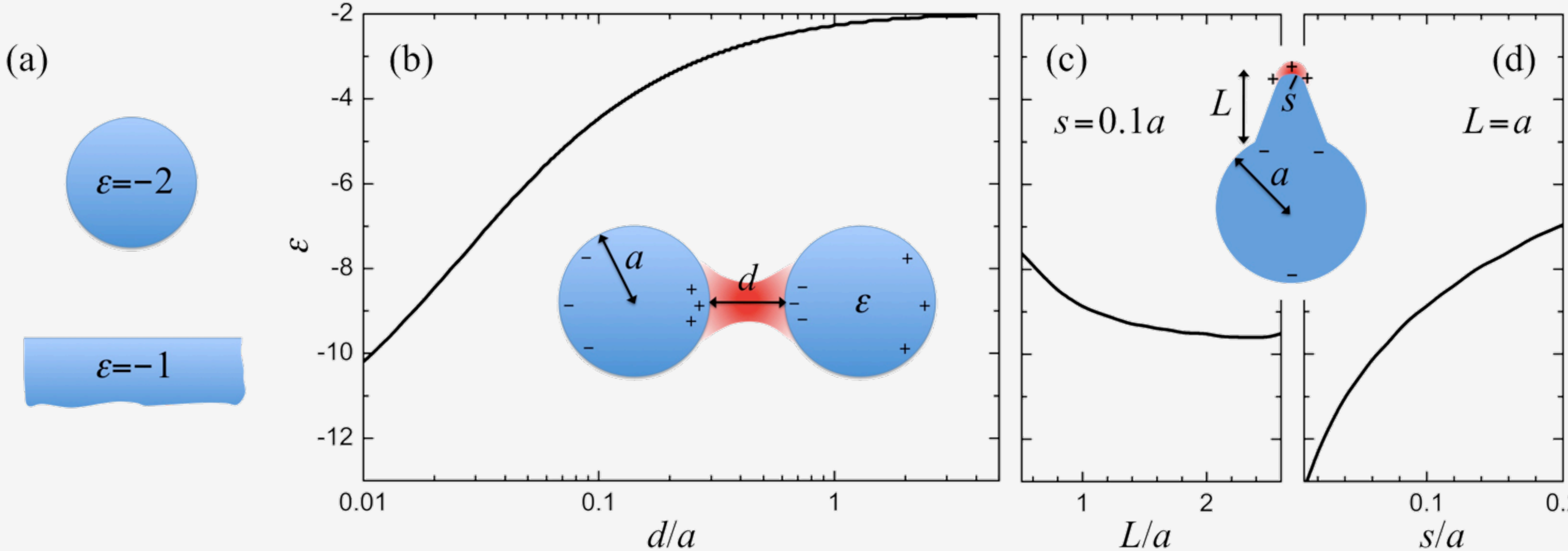
Myroshnychenko *et al*., Advanced Materials (2008)

Introduction to plasmons

Surface plasmons are surface waves involving collective electron motion and propagating on metal surfaces or localized in metal (nano)structures (e.g., nanoparticles), where they couple efficiently to light, they produce strong confinement of the electromagnetic field (size << wavelength) …

Why do we need metals?

Plasmons in the long wavelength limit (Poisson equation) are scale-invariant, and therefore, they exist for structures down to a few nm.



Localized excitations require negative permittivity

Álvarez-Puebla *et al*., J. Phys. Chem. Lettt. (2010)

Why do we need metals?

Maxwell equations …

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},\omega)e^{-i\omega t}+\mathbf{E}^{*}(\mathbf{r},\omega)e^{i\omega t}$$

$$\nabla\cdot\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)=0 \qquad \nabla\times\mathbf{E}(\mathbf{r},\omega)=i\frac{\omega}{c}\mathbf{B}(\mathbf{r},\omega)$$

$$\nabla\cdot\mathbf{B}(\mathbf{r},\omega)=0 \qquad \nabla\times\mathbf{H}(\mathbf{r},\omega)=-i\frac{\omega}{c}\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)$$

Why do we need metals?

Maxwell equations for small particles

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},\omega)e^{-i\omega t}+\mathbf{E}^{*}(\mathbf{r},\omega)e^{i\omega t}$$

$$\nabla\cdot\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)=0 \qquad \nabla\times\mathbf{E}(\mathbf{r},\omega)=0$$

$$\nabla\cdot\mathbf{B}(\mathbf{r},\omega)=0 \qquad \nabla\times\mathbf{H}(\mathbf{r},\omega)=0$$

Electricity and magnetism are decoupled
in the long-wavelength ($c\rightarrow\infty$) limit*

*Except in regions of very index of refraction, $|n|a\sim\lambda$

Why do we need metals?

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Maxwell equations for small particles

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},\omega)e^{-i\omega t} + \mathbf{E}^*(\mathbf{r},\omega)e^{i\omega t}$$

$$\nabla \cdot \varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) = 0 \qquad \nabla \times \mathbf{E}(\mathbf{r},\omega) = 0$$

$$\mathbf{H} = 0$$

Why do we need metals?

Maxwell equations for small particles

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},\omega)e^{-i\omega t}+\mathbf{E}^*(\mathbf{r},\omega)e^{i\omega t}$$

$$\nabla\cdot\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)=0 \qquad \mathbf{E}(\mathbf{r},\omega)=-\nabla\phi(\mathbf{r},\omega)$$

$$\nabla\cdot\varepsilon(\mathbf{r},\omega)\nabla\phi(\mathbf{r},\omega)=0$$

The Poisson equation also describes stationary heat transport:

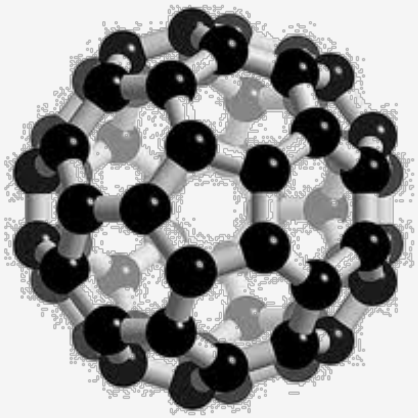
ε → *k*, thermal conductivity
ϕ → temperature

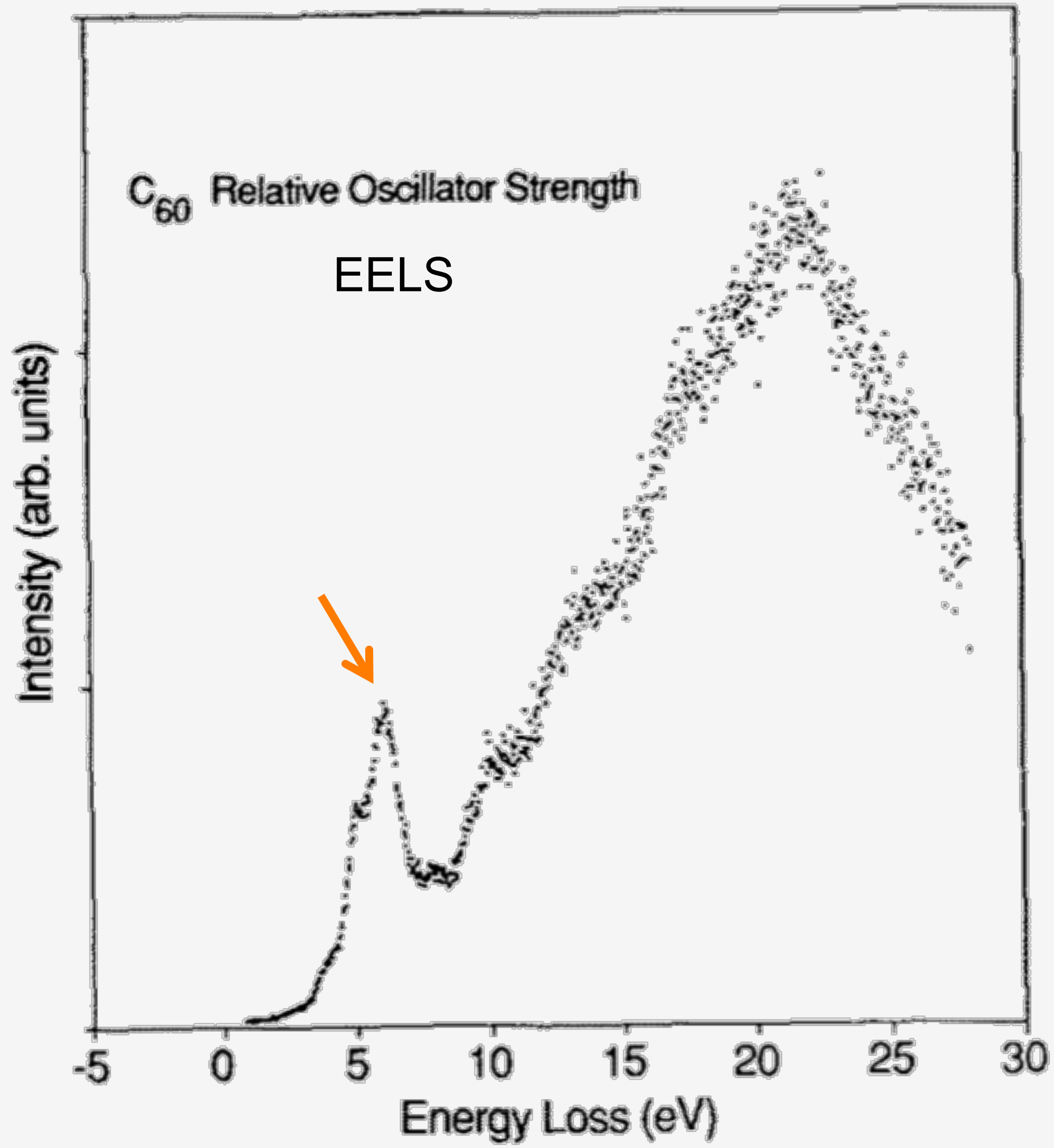
Thermodynamics:

- flow towards lower temperature regions → k>0
- absence of trapped thermal energy

But … do we really need metals?

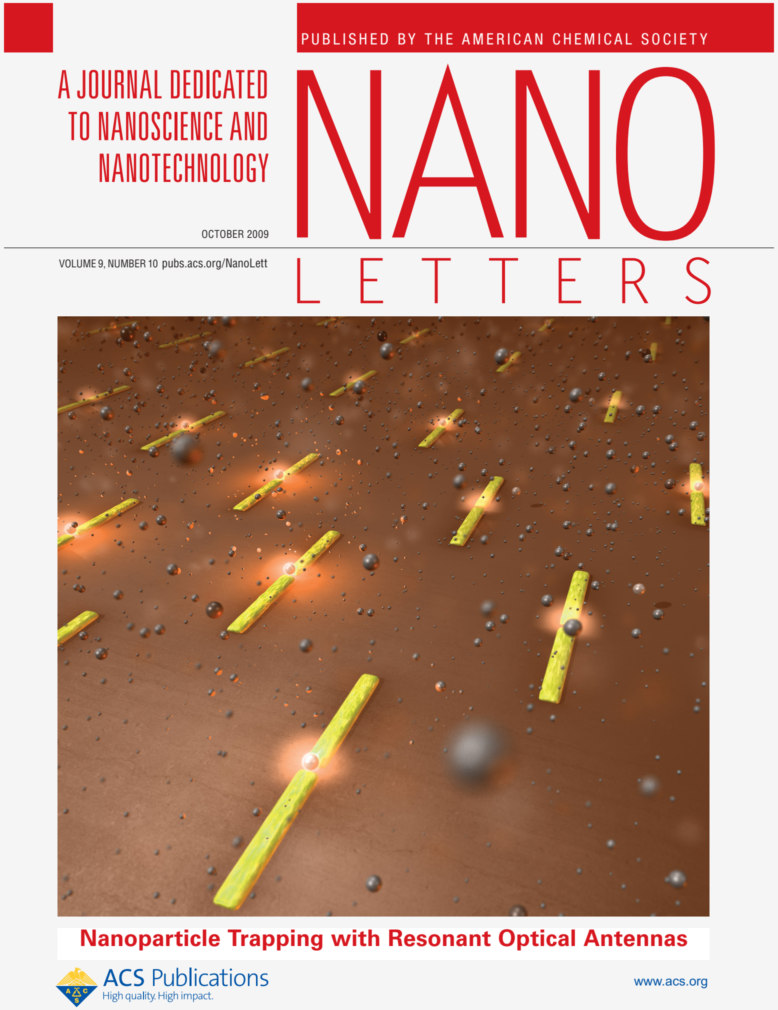
C_{60}

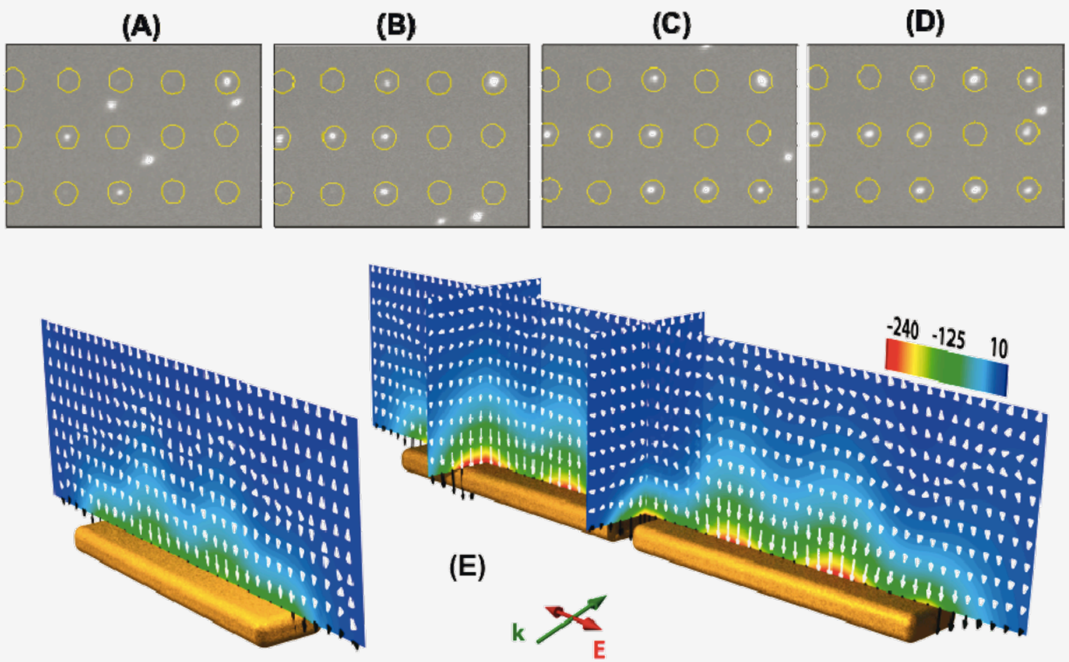




Keller and Coplan, Chem. Phys. Lett. (1992)

Optical trapping - nanotweezers



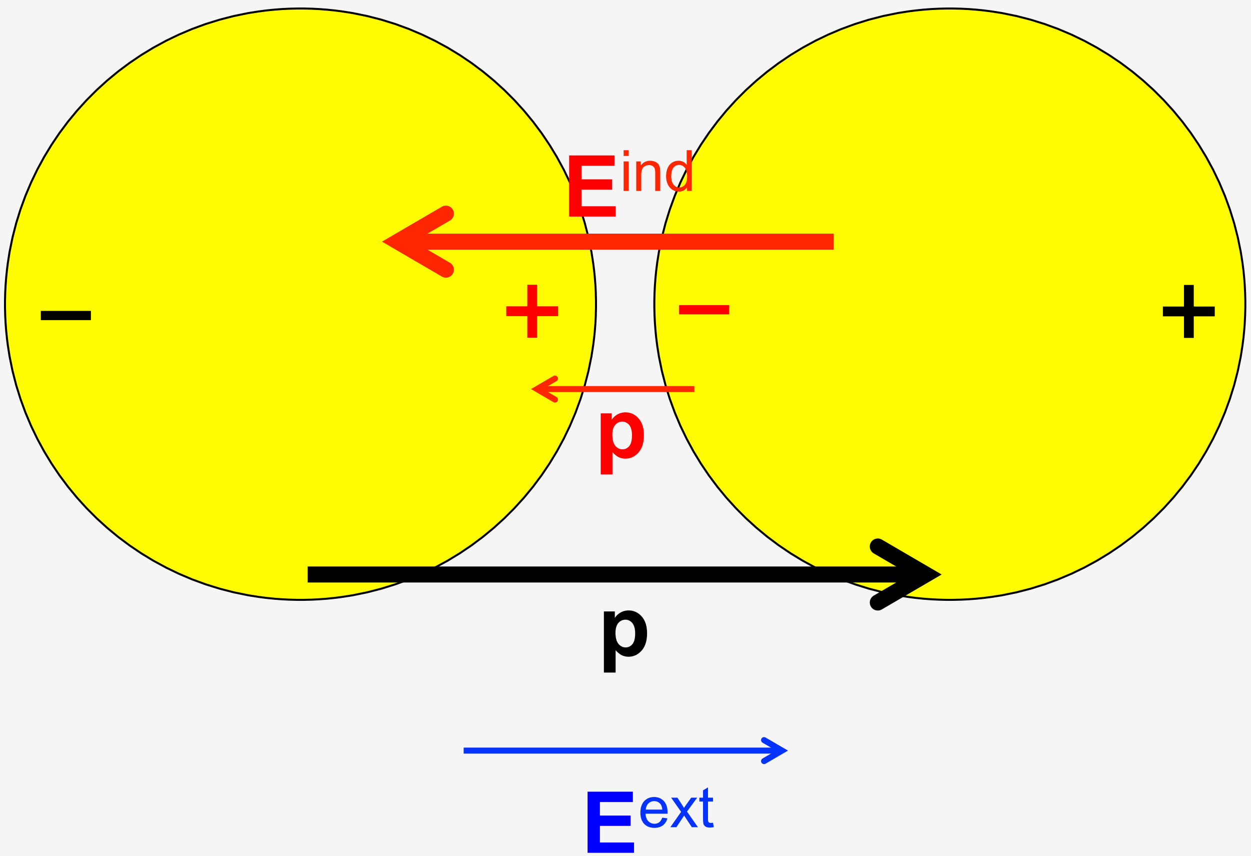


Righini *et al*. (2009)

Introduction to plasmons

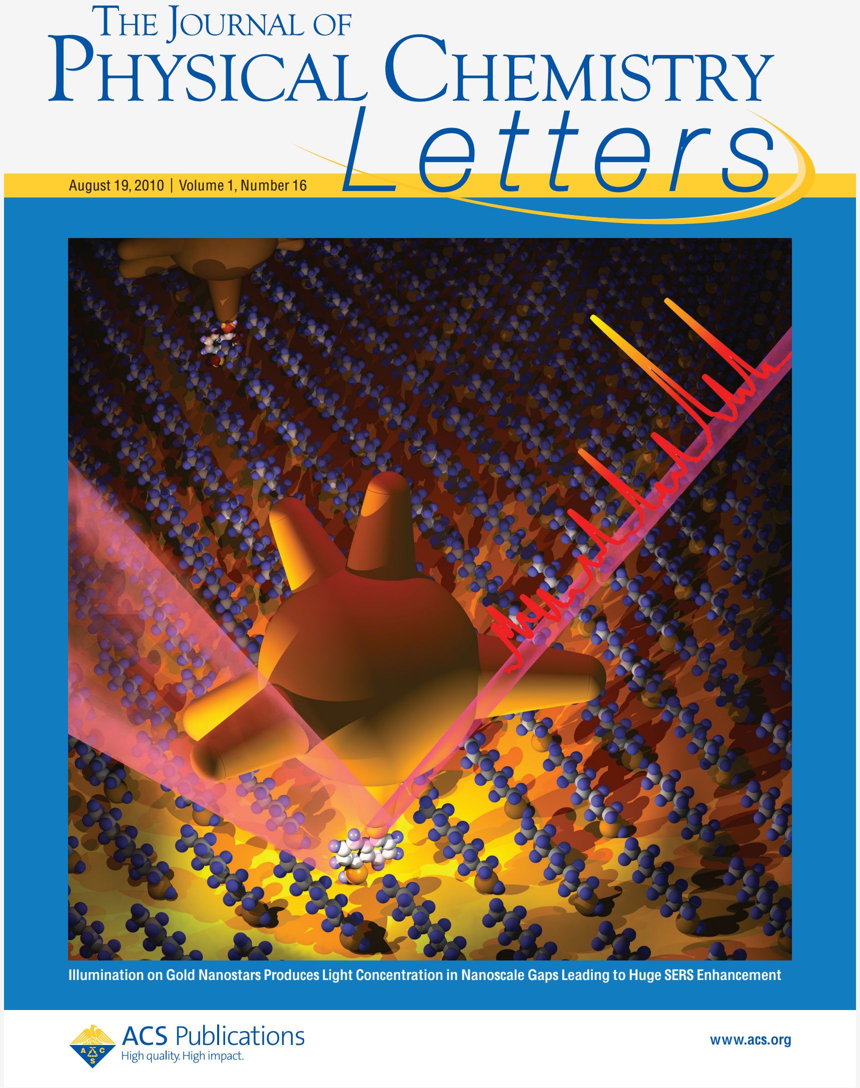
Surface plasmons are surface waves involving collective electron motion and propagating on metal surfaces or localized in metal (nano)structures (e.g., nanoparticles), where they couple efficiently to light, they produce strong confinement of the electromagnetic field (size << wavelength), and they generate huge enhancement of the optical electric-field intensity.

Optical field enhancement through plasmons



Charge neutrality → strong coupling to light though **p**, strong enhancement in the gap

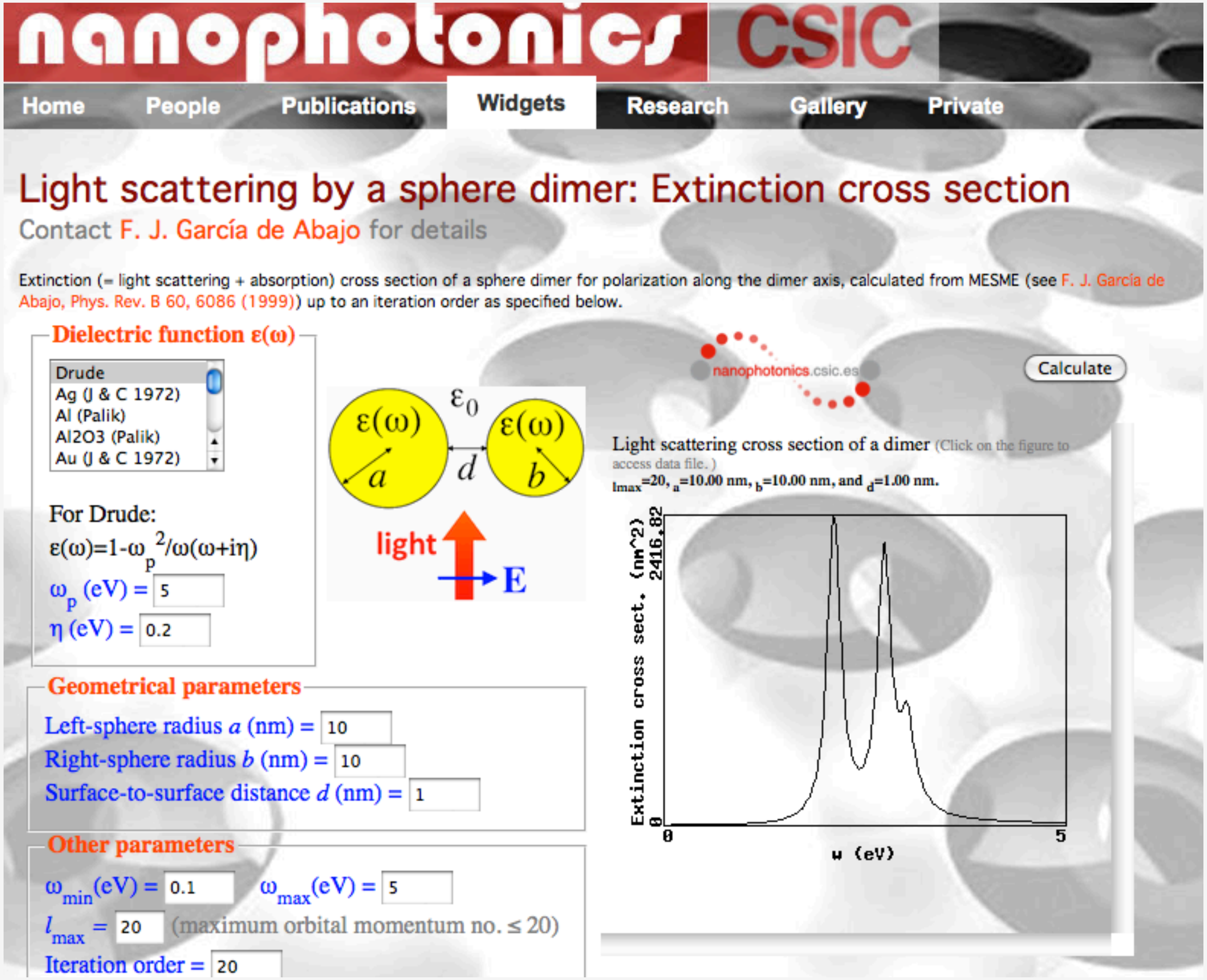
Controlled 10^{10} SERS enhancement → 10^5 intensity enhancement



Álvarez-Puebla, Liz-Marzán, G. de Abajo, JACS (2009), JPCL (2010) nanophotonics.csic.es

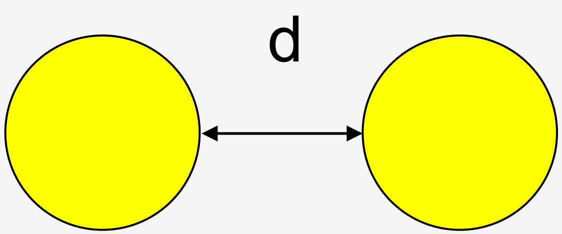
Plasmon simulation has become
a simple task in most systems of current interest,
for example to understand confined
gap plasmons in nanoparticle dimers
(e.g., check the widgets of our website for
this and other applications).

Dimer widget at http://www.nanophotonics.es

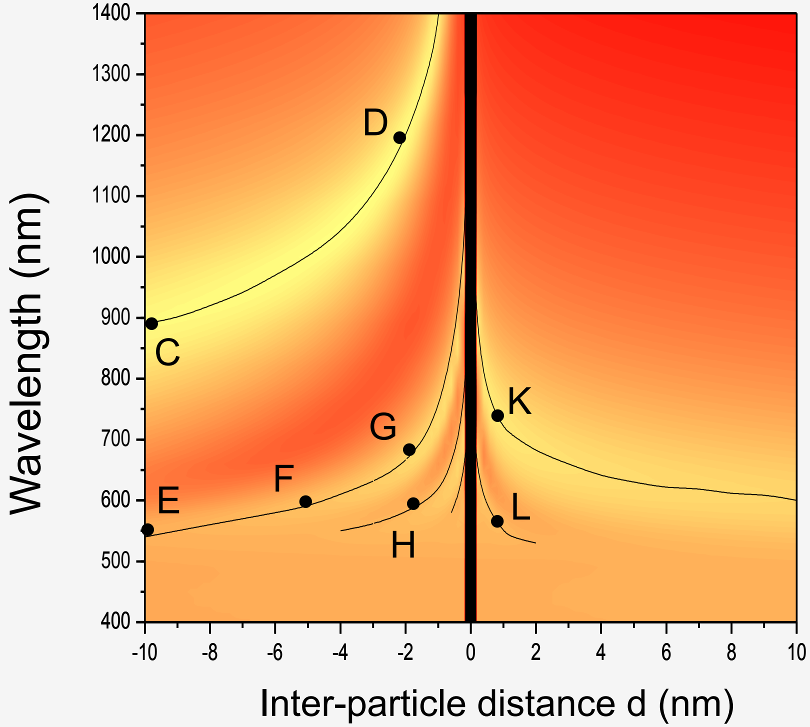


Transition between touching and non-touching

Particle dimer: transition from touching to non-touching



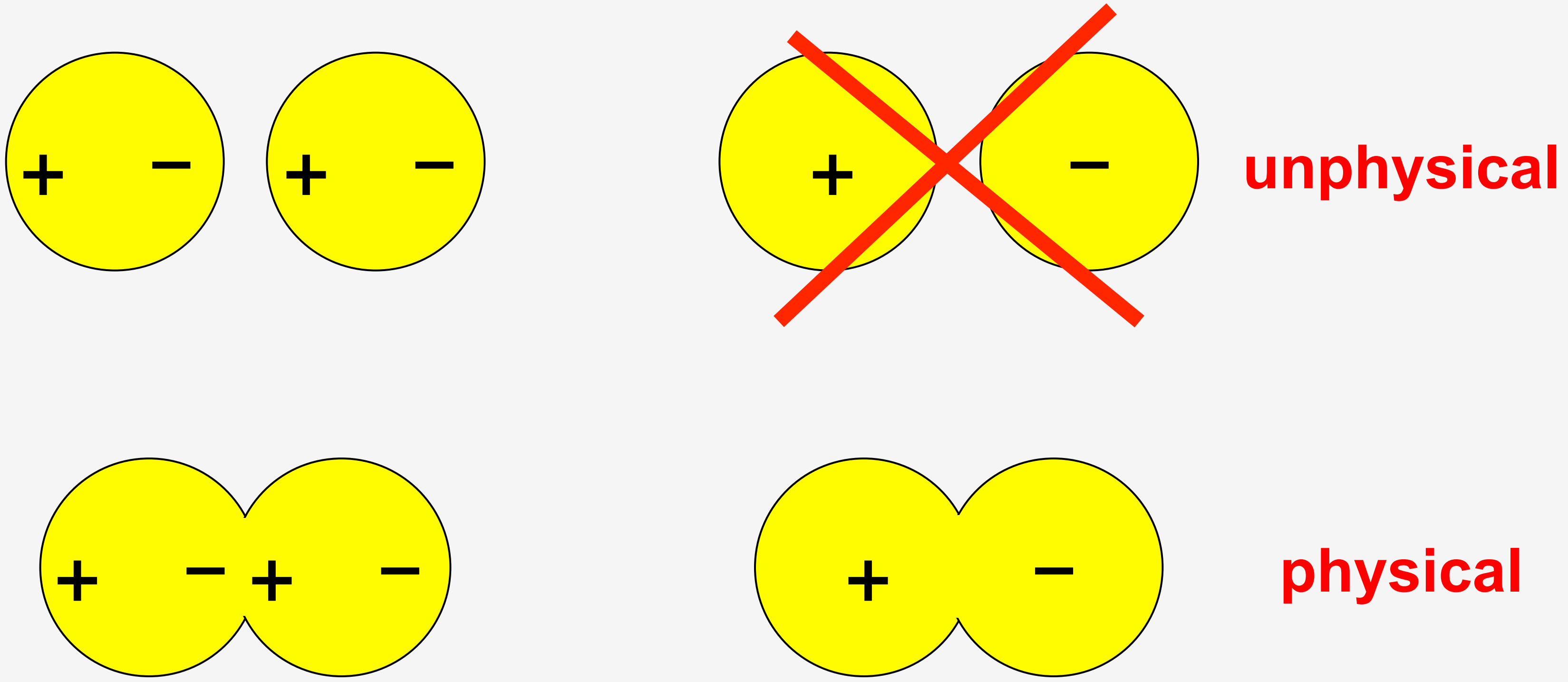
120-nm Au particles



Romero *et al.*, Optics Express (2006)

Transition between touching and non-touching

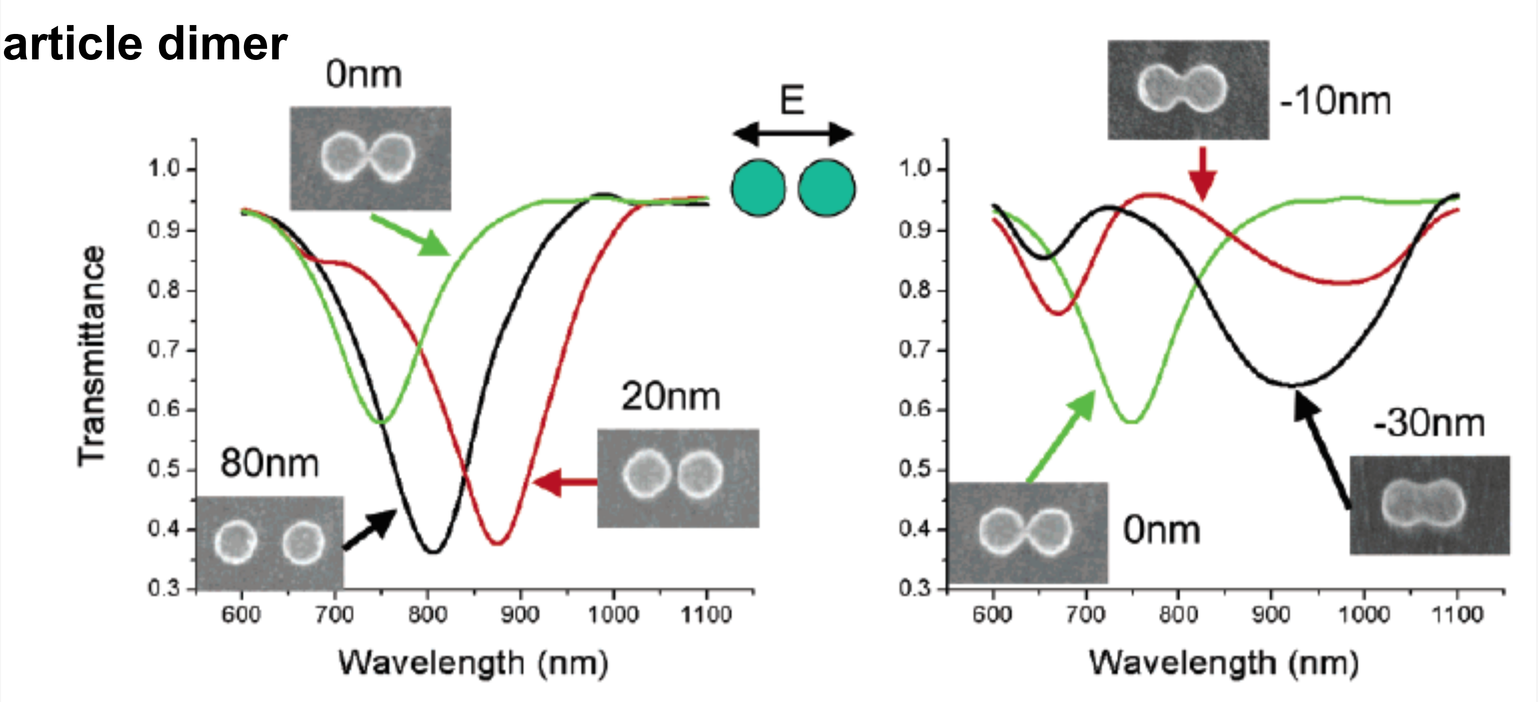
Particle dimer: transition from touching to non-touching

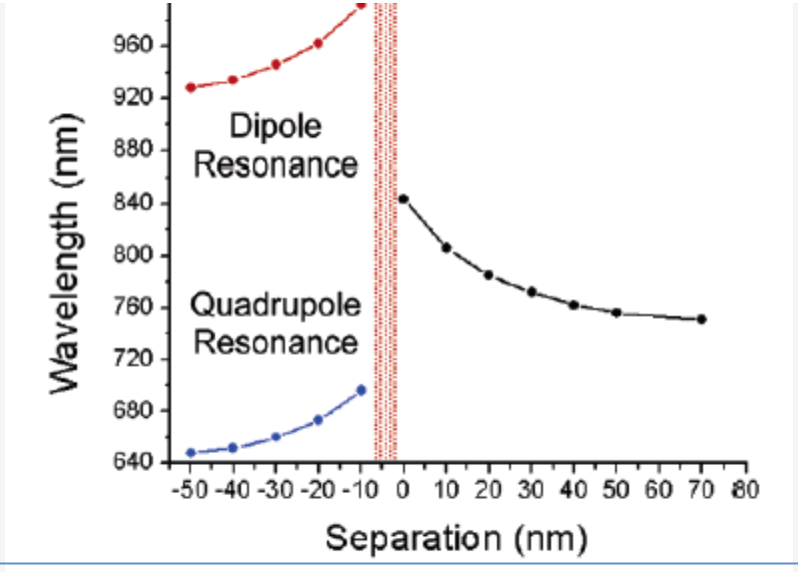


Romero *et al.*, Optics Express (2006)

Transition between touching and non-touching

Particle dimer





Atay *et al.*,
Nano Letters (2004)

Introduction to plasmons

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Plasmons can be imaged with nanometer precision using electron microscopes via EELS and cathodoluminescence.

See García de Abajo, Rev. Mod. Phys. **82**, 209 (2010).

Plasmon modes in SRRs imaged by EELS

PRL 105, 255501 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

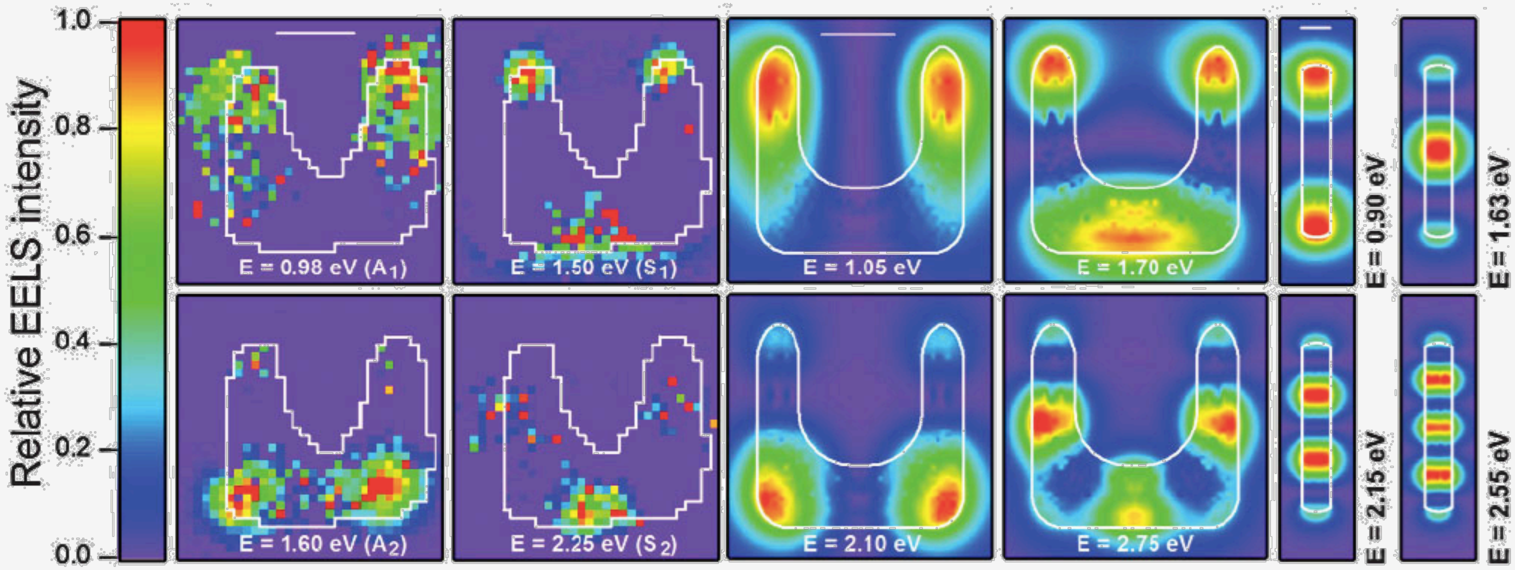
Spectral Imaging of Individual Split-Ring Resonators

Guillaume Boudarham,[1] Nils Feth,[2] Viktor Myroshnychenko,[3] Stefan Linden,[2,4] Javier García de Abajo,[3]
Martin Wegener,[2,4,5] and Mathieu Kociak[1,*]

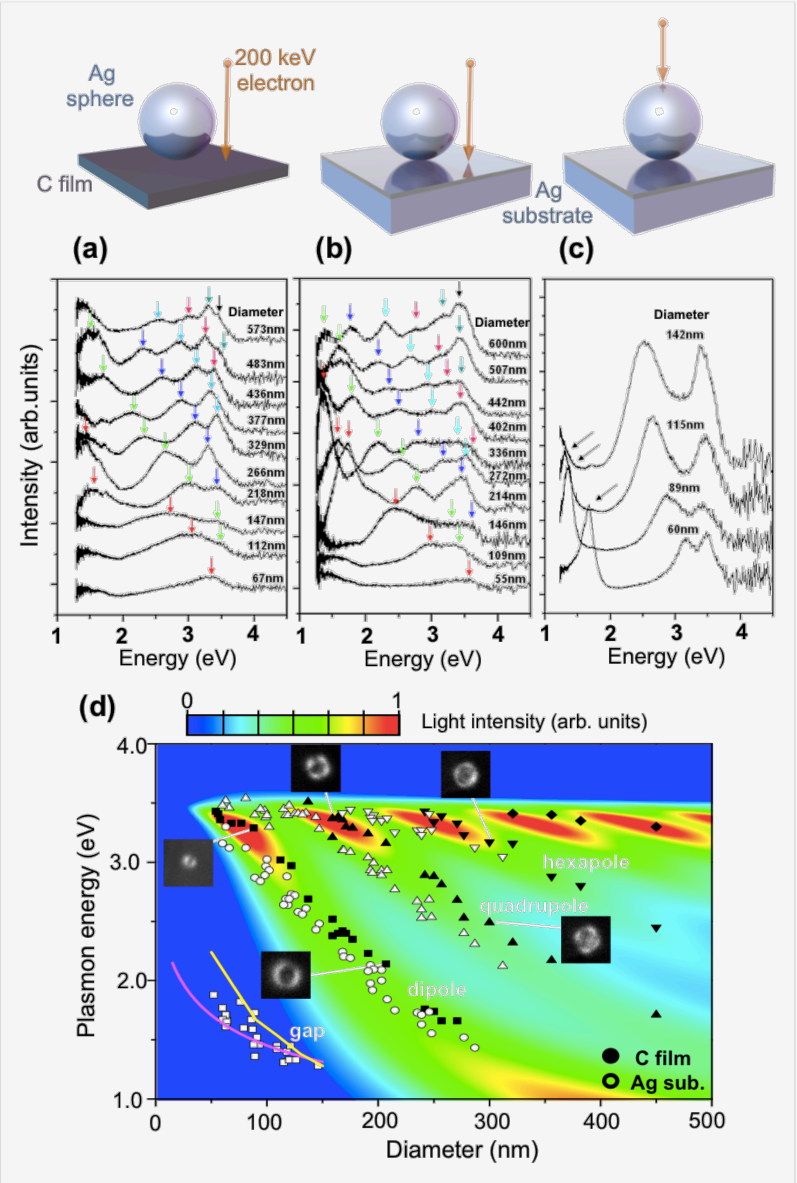
[1]*Laboratoire de Physique des Solides CNRS/UMR8502, Bâtiment 510, Univ. Paris-Sud, Orsay, 91405, France*
[2]*Institut für Angewandte Physik, Universität Karlsruhe, Wolfgang-Gaede-Strasse 1, 76131 Karlsruhe, Germany*
[3]*Instituto de Óptica, CSIC, Serrano 121, 28006 Madrid, Spain*
[4]*Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*
[5]*DFG-Center for Functional Nanostructures, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*

Experiment

Theory



Mie and gap plasmons imaged by cathodoluminescence

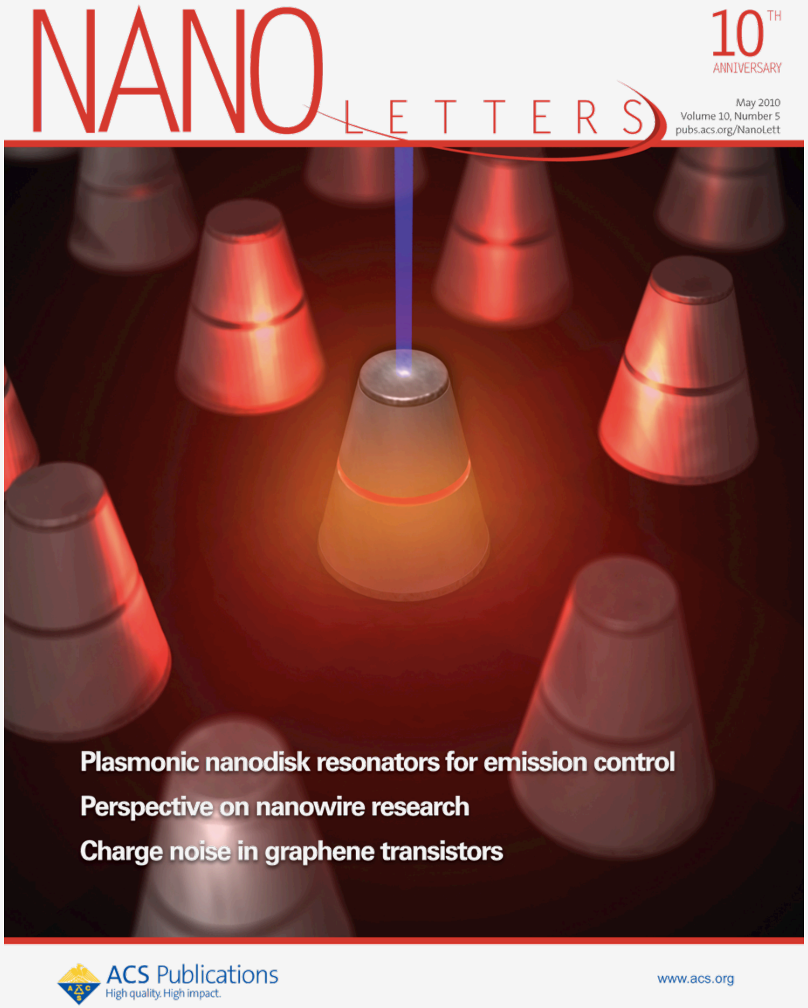


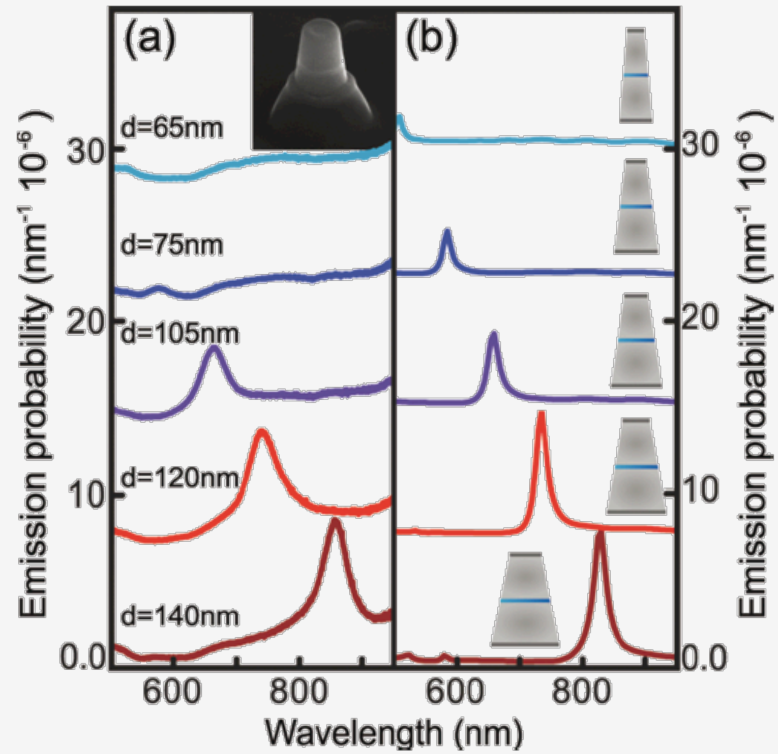
Yamamoto *et al*., Nano Letters (2010)

Optimally coupling to confined plasmons

Ultrasmall Mode Volume Plasmonic Nanodisk Resonators

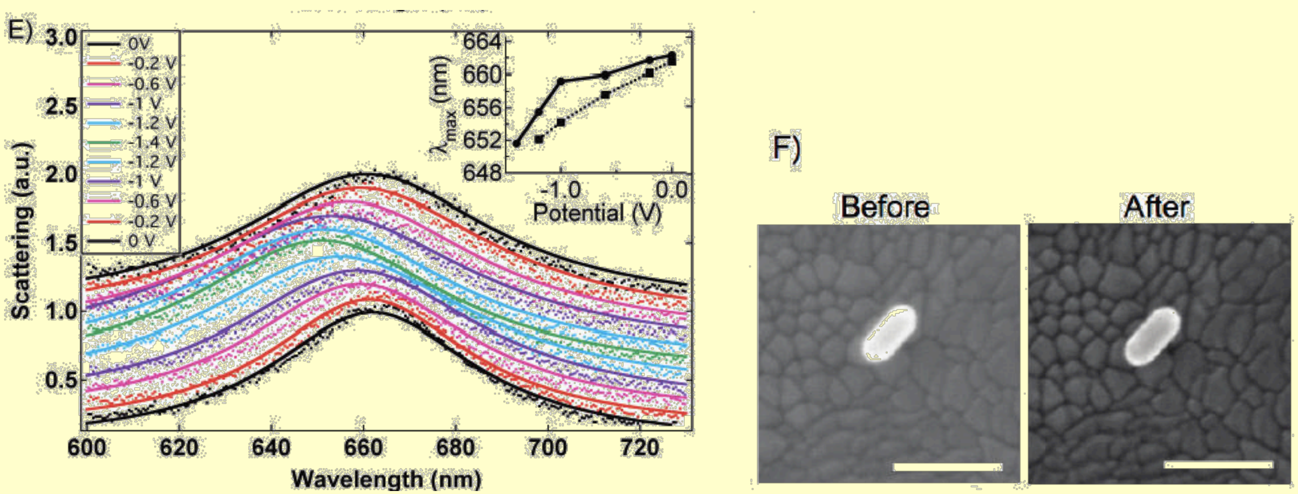
Martin Kuttge,*,† F. Javier García de Abajo,† and Albert Polman†





Appealing properties of plasmons

- Light concentration: plasmon size << wavelength
- Field enhancement: induced field >> external field
- Fast tunability: electric doping

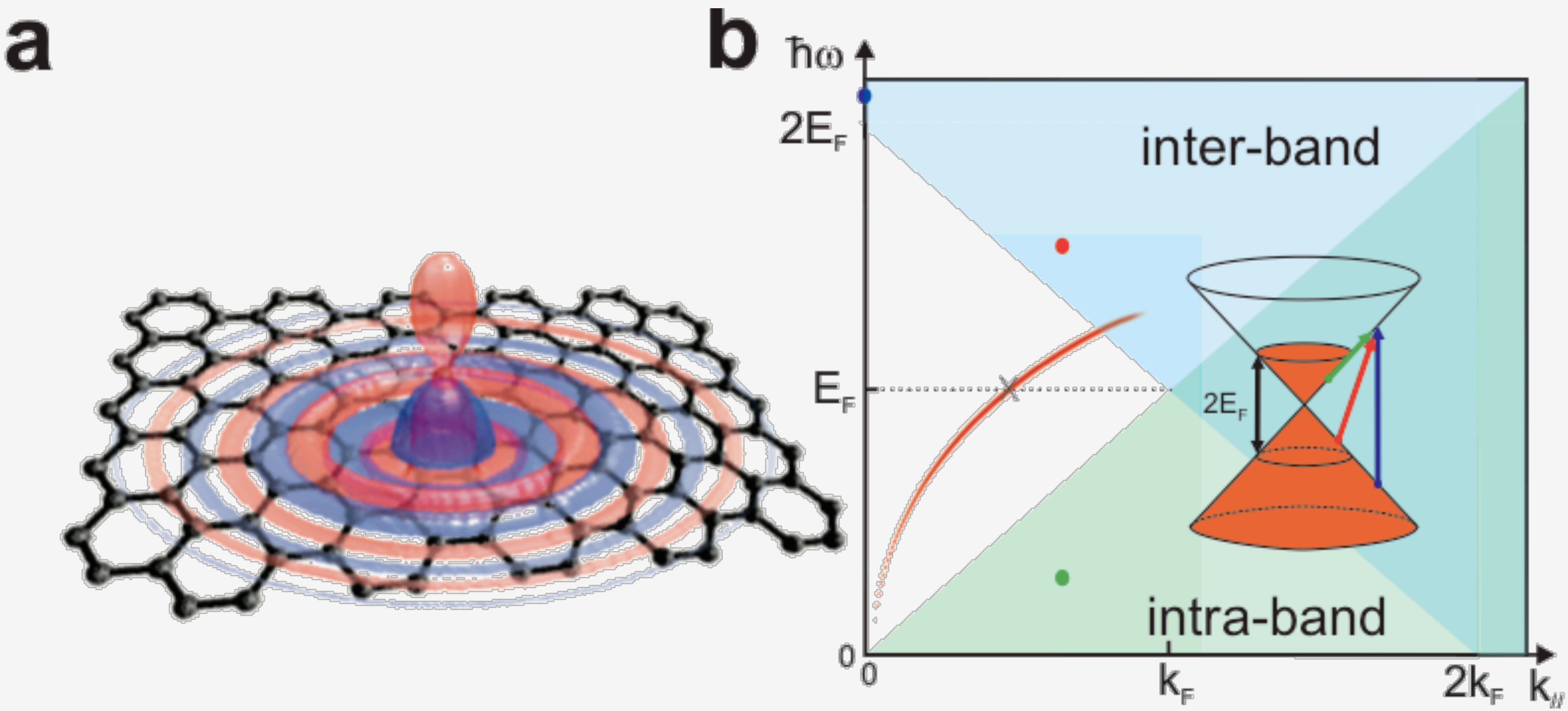


Novo, Funston, Gooding, Mulvaney, JACS (2009)

Graphene is a tunable plasmonic material
that produces unprecedented confinement
and strong light-matter interaction
in a robust, solid-state environment

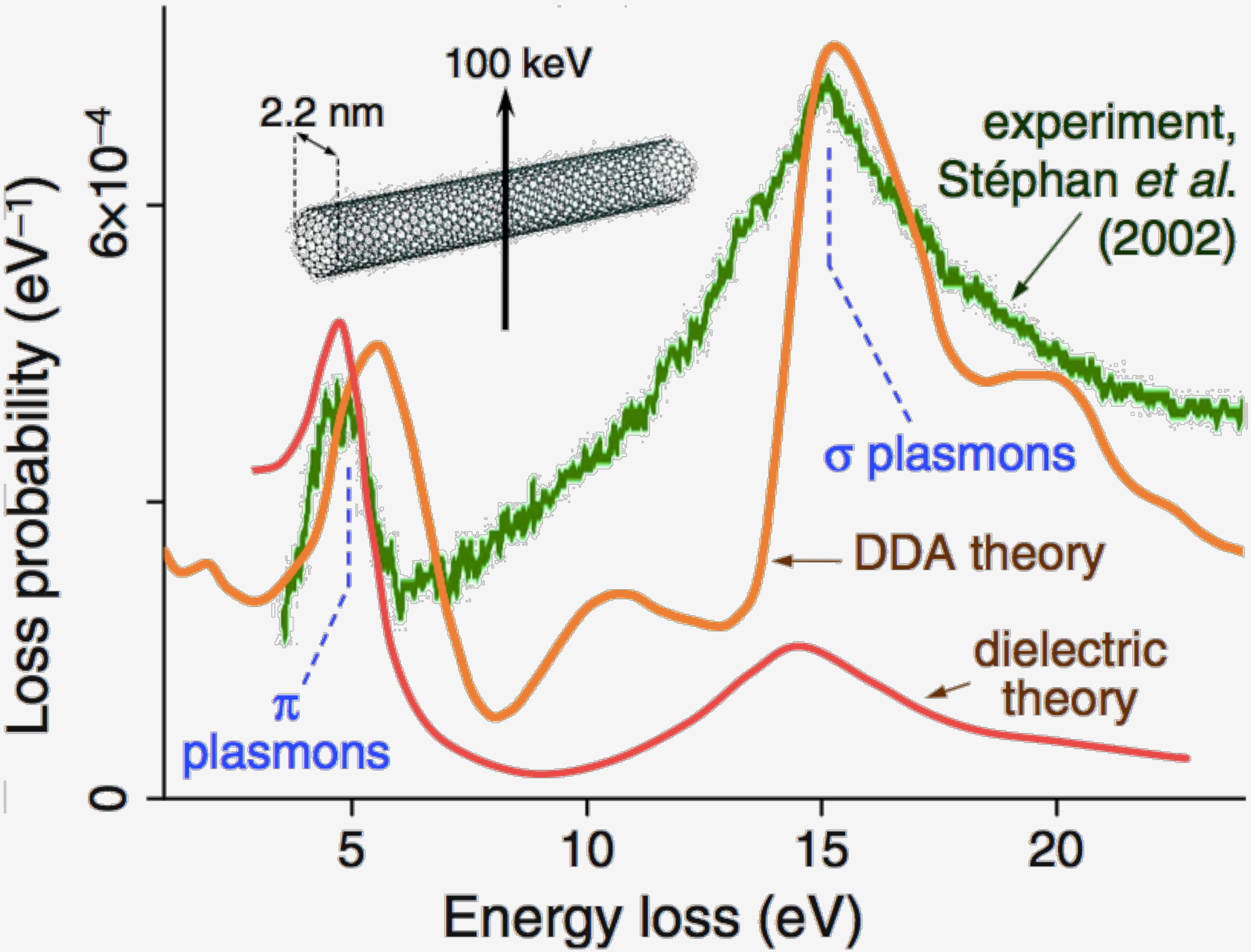
Koppens, Chang, García de Abajo,
Nano Letters **11**, 3370 (2011)

Graphene plasmons



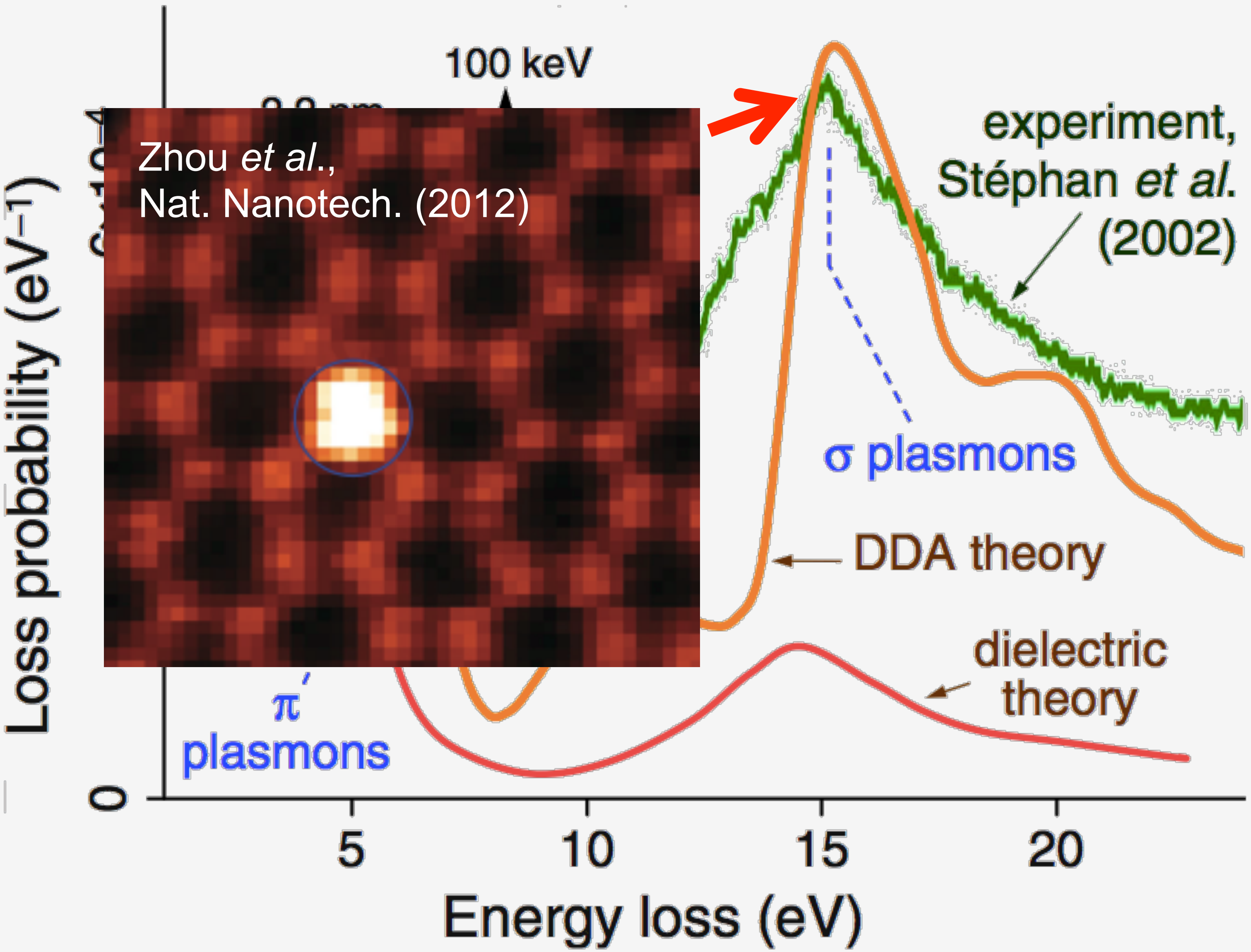
Koppens *et al.*, Nano Lett. (2011)

Graphene plasmons



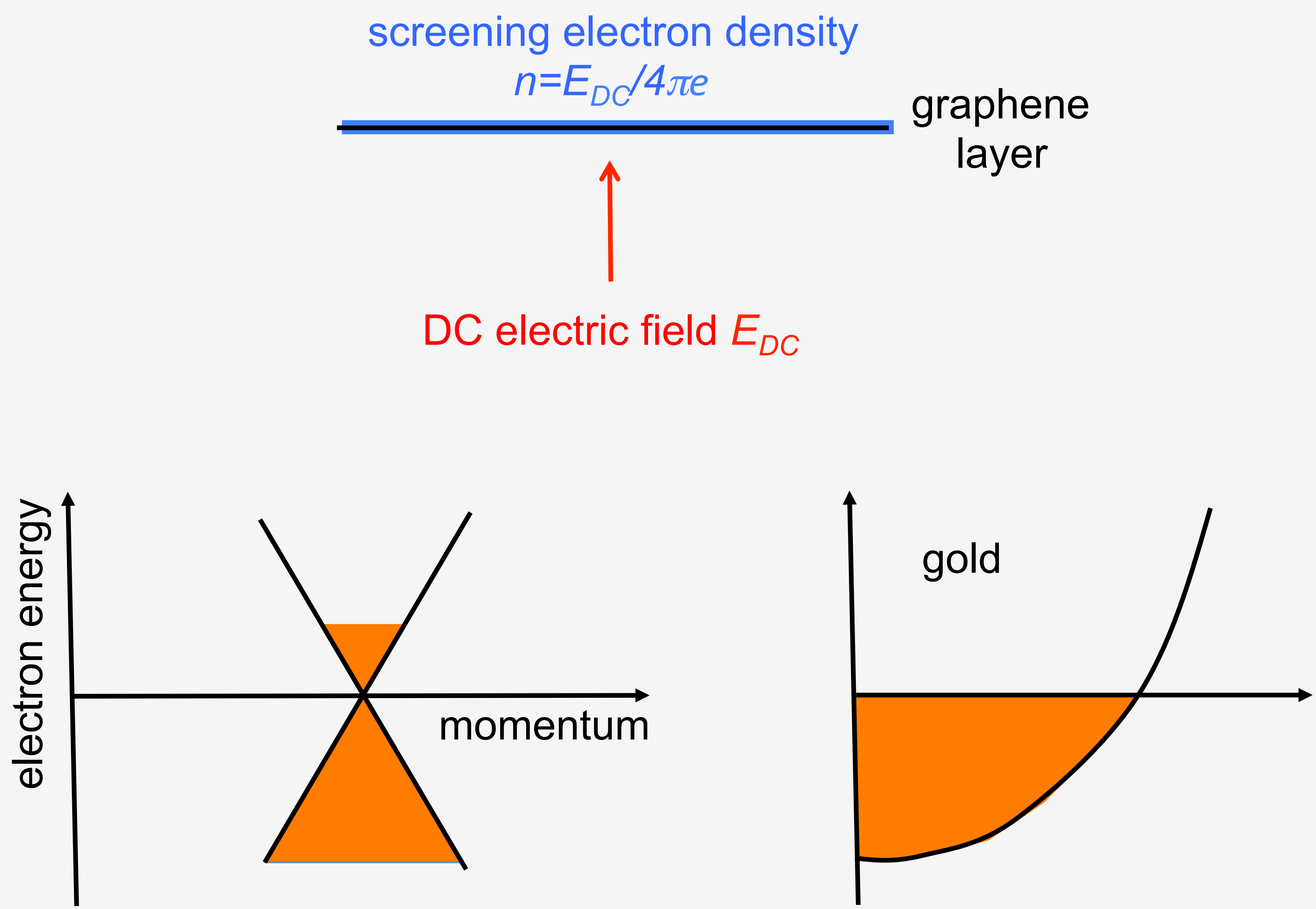
García de Abajo, Rev. Mod. Phys. (2010)

Graphene plasmons

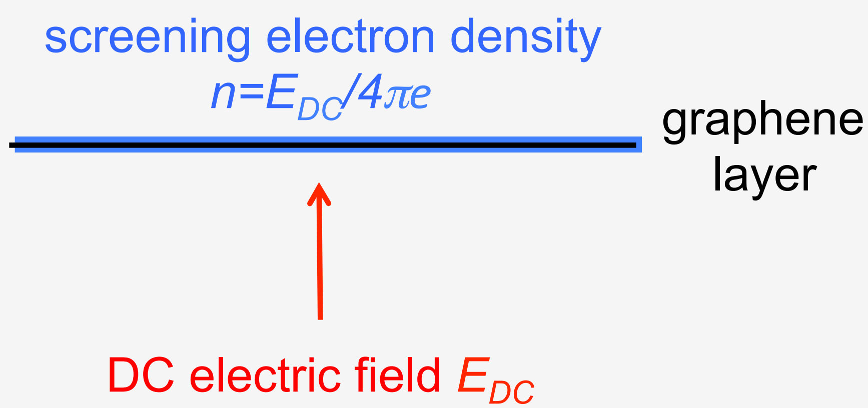


García de Abajo, Rev. Mod. Phys. (2010)

Electrostatic doping of graphene

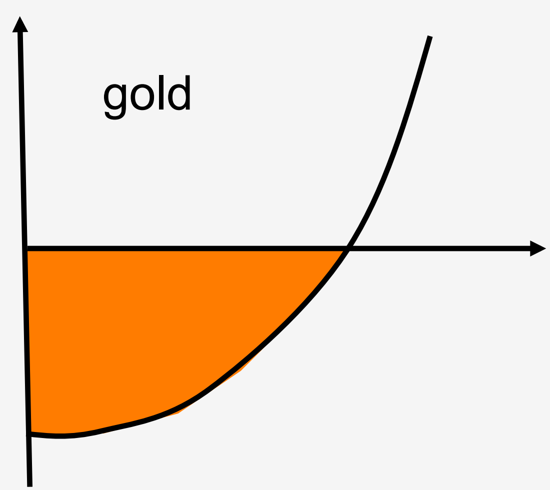


Electrostatic doping of graphene

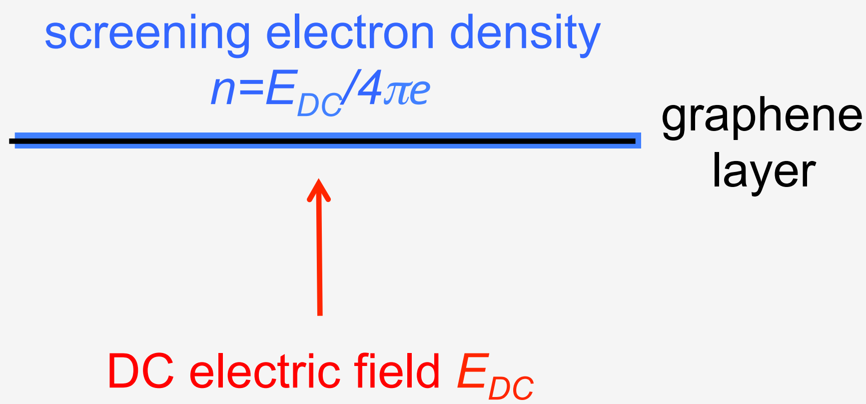


$$\sigma(\omega) = \frac{ie^2 v_F}{\pi^{3/2}\hbar} \frac{\sqrt{n}}{\left(\omega + i\tau^{-1}\right)}$$

$$v_F = 10^6\, m/s$$



Electrostatic doping of graphene



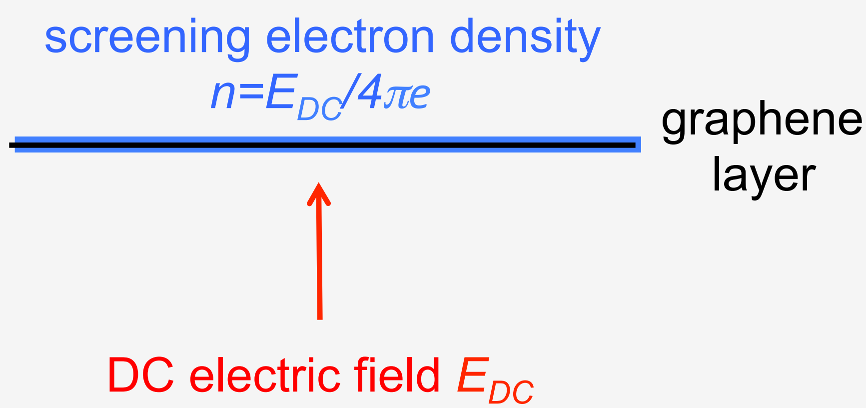
$$\sigma(\omega) = \frac{ie^2 v_F}{\pi^{3/2}\hbar} \frac{\sqrt{n}}{\left(\omega + i\tau^{-1}\right)}$$

$$v_F = 10^6 m/s$$

gold

$$\varepsilon(\omega) = 1 - \frac{4\pi e^2}{m} \frac{n}{\omega\left(\omega + i\tau^{-1}\right)}$$

Electrostatic doping of graphene

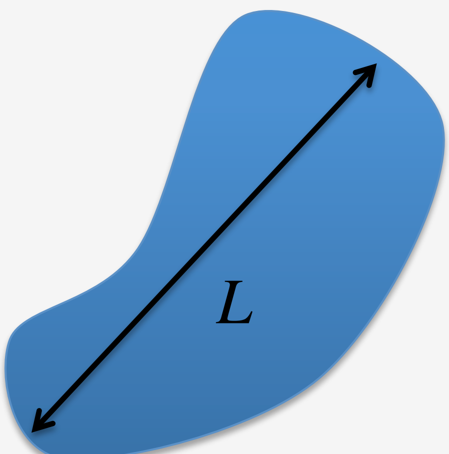


$$\sigma(\omega) = \frac{ie^2 v_F}{\pi^{3/2}\hbar} \frac{\sqrt{n}}{\left(\omega + i\tau^{-1}\right)}$$

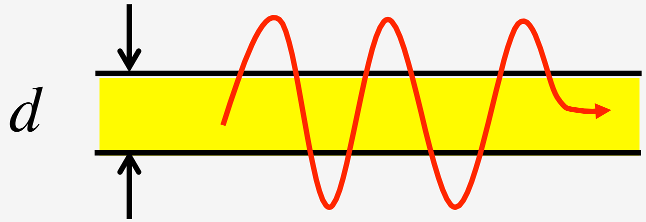
$$\omega_p = C \frac{E_{DC}^{1/4}}{\sqrt{L}}$$

$$a << \lambda$$

$$v_F = 10^6 m/s$$



Graphene vs gold



$$\varepsilon(\omega) \approx \frac{4\pi i \sigma}{\omega d}$$

$$\lambda_{sp} = \frac{-2\pi i \sigma}{c} \lambda$$

1 ML of gold
(*d*=0.24 nm)

$$\sigma \approx \frac{\omega_p^2 d}{4\pi} \frac{i}{\omega + i\tau^{-1}}$$

$$\tau = 1.0 \times 10^{-14}\,\mathrm{s}$$

$$\hbar\omega = 0.5\ \mathrm{eV} \ \rightarrow \lambda_{sp} = 240\ \mathrm{nm}$$

graphene
(*d*=0.3 nm)

$$\sigma \approx \frac{e^2 E_F}{\hbar^2 \pi} \frac{i}{\omega + i\tau^{-1}}$$

$\tau = 0$ … eV

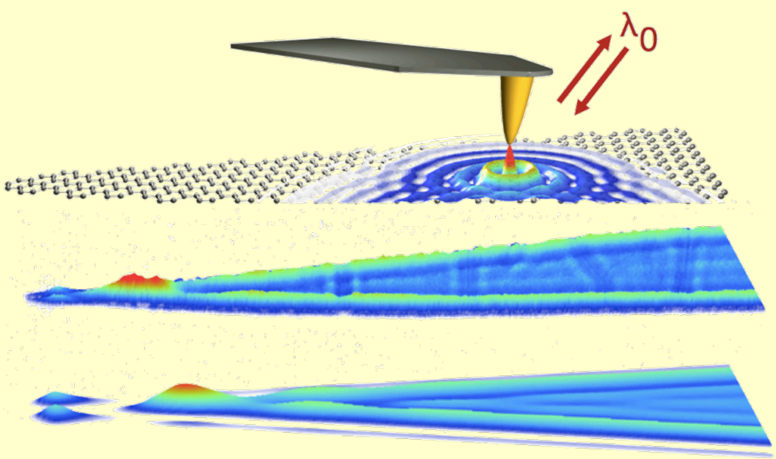
$\hbar$…

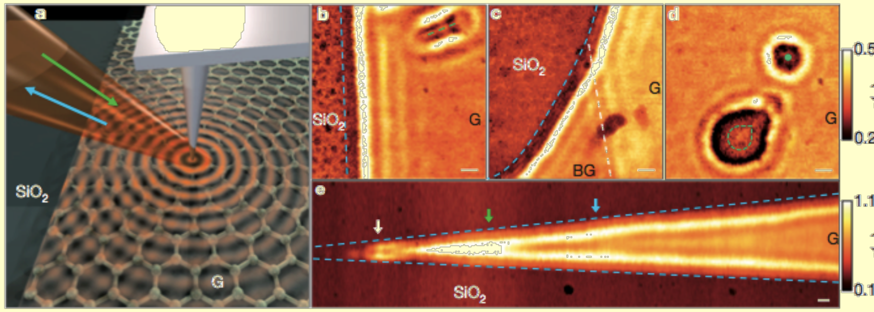
$$\lambda_{sp} = 2\alpha \frac{E_F}{\hbar\omega} \lambda_0$$

$$\text{decay length: } \sim (\omega\tau / 2\pi)\lambda_{sp}$$

$1.2\lambda_{sp}$ $\qquad$ $60\lambda_{sp}$

Experimental demonstration of
spatial mapping and
electrical tunability
of graphene plasmons

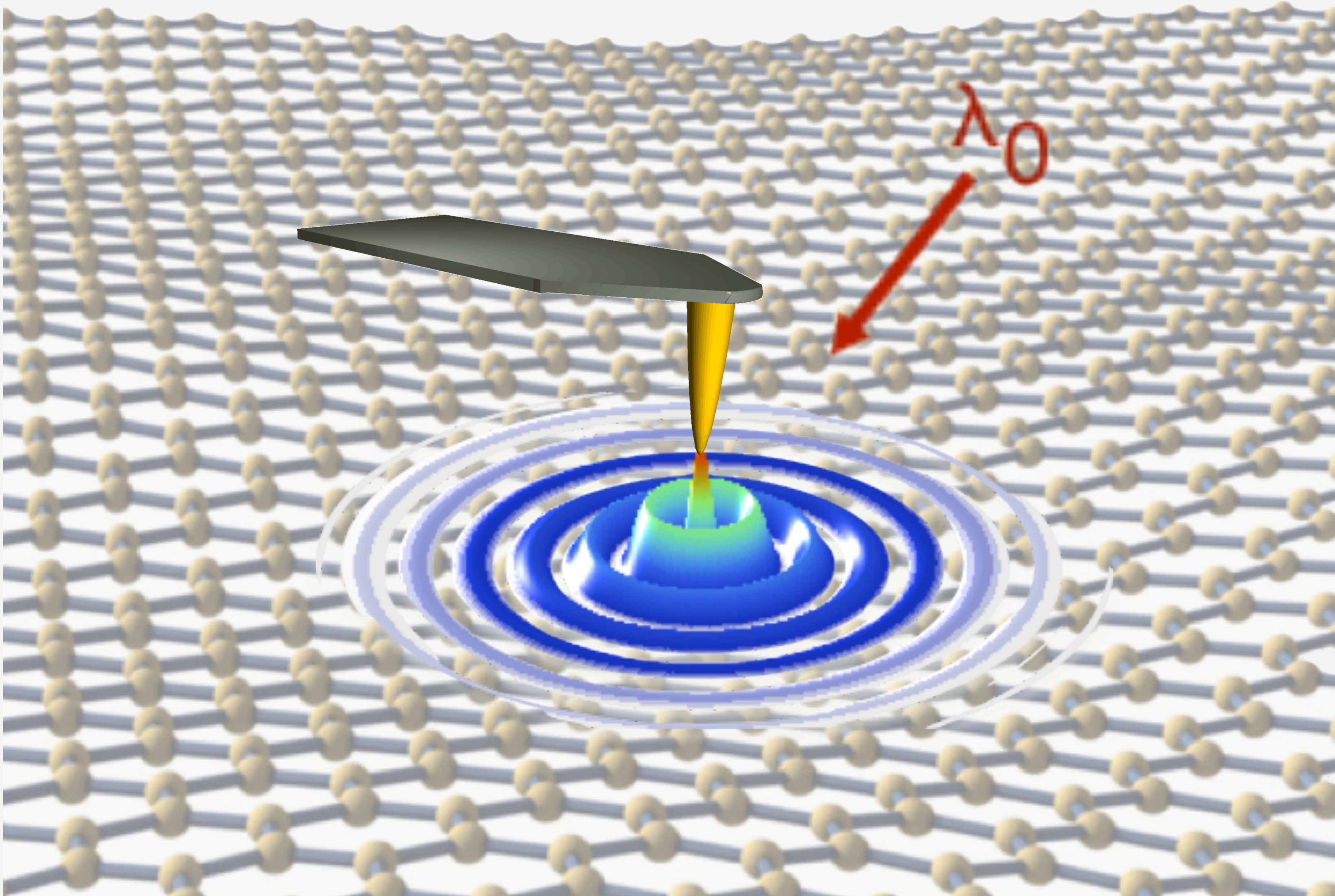




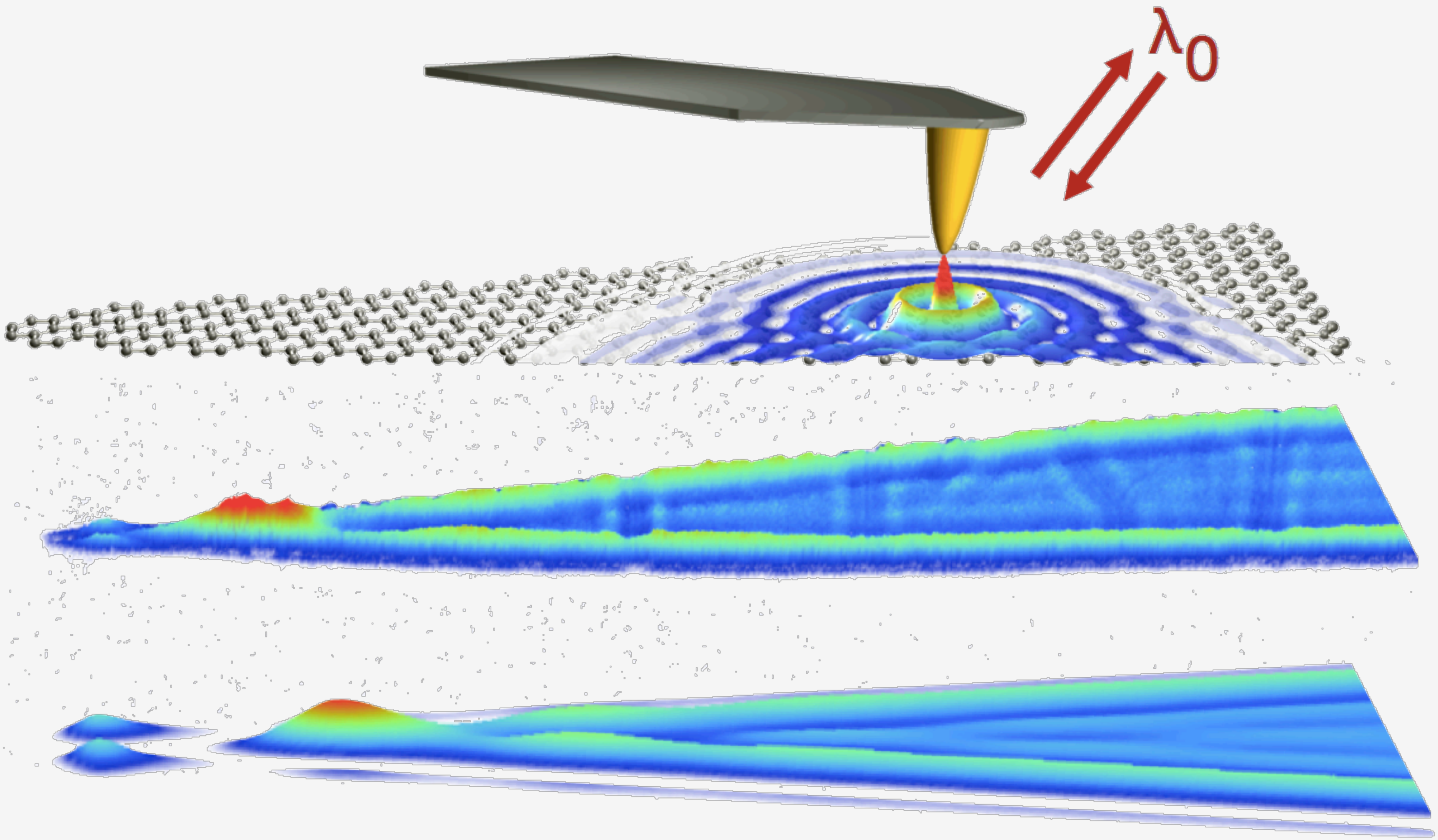
Chen *et al.*, Nature (2012)

Basov's group Fei *et al.*, Nature (2012)

Experimental mapping of graphene plasmons



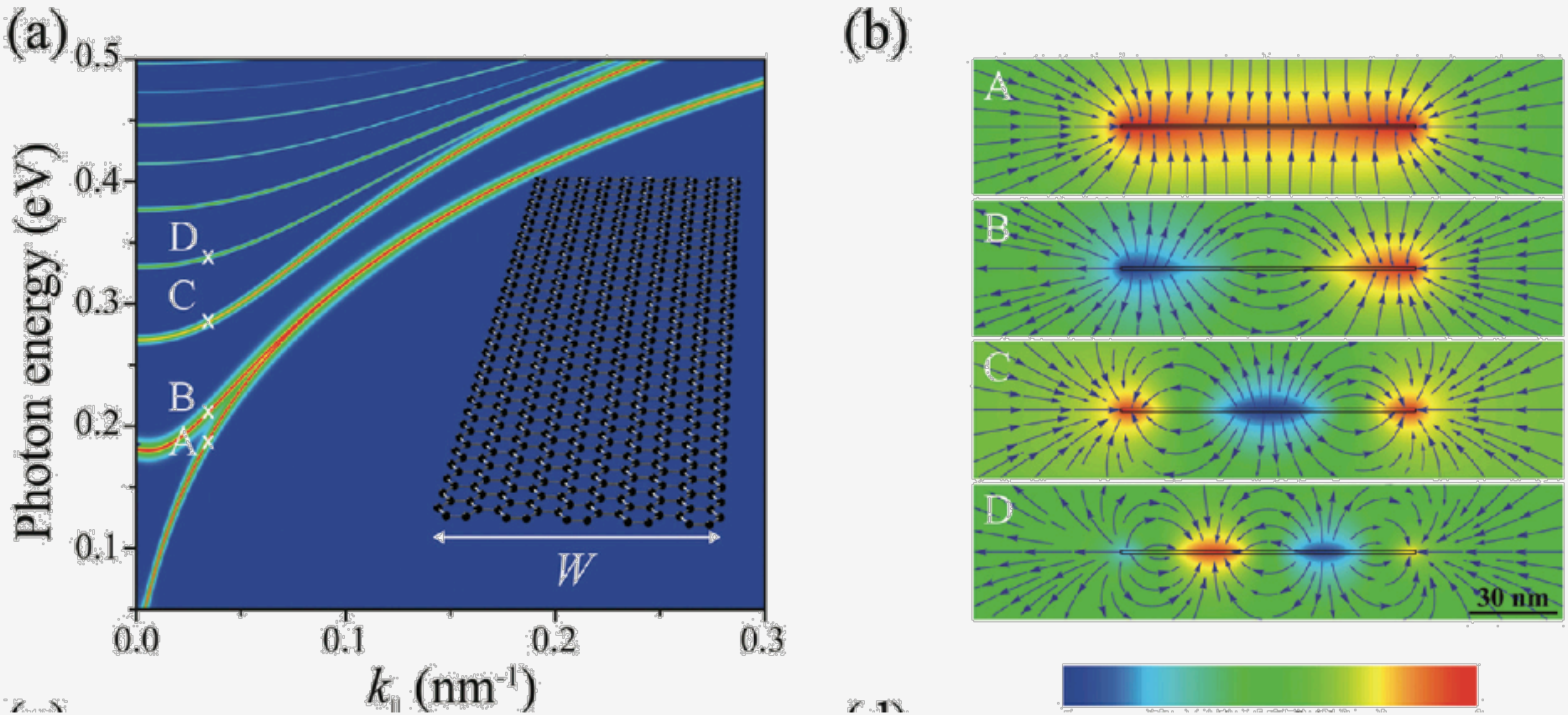
Experimental mapping of graphene plasmons



Chen *et al*., Nature (2012)

Also, Basov's group Fei *et al*., Nature (2012)

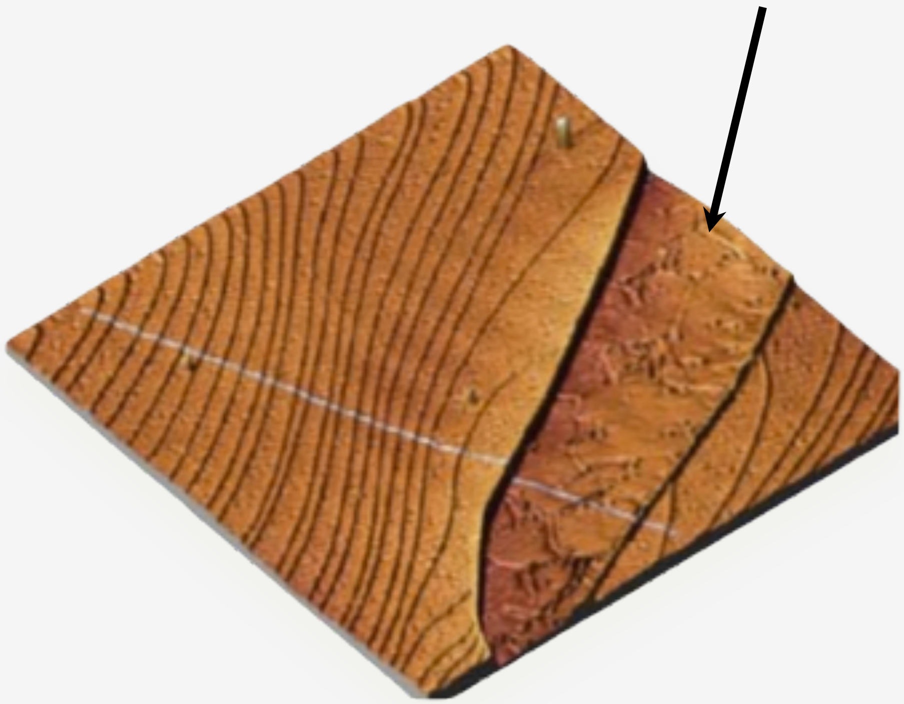
Plasmons in graphene ribbons

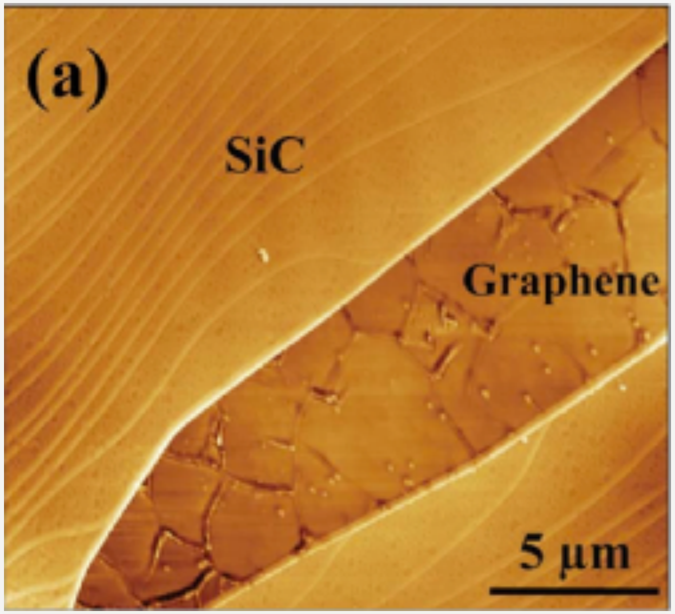


Christensen *et al.*, ACS Nano (2012)

Experimental mapping of graphene plasmons

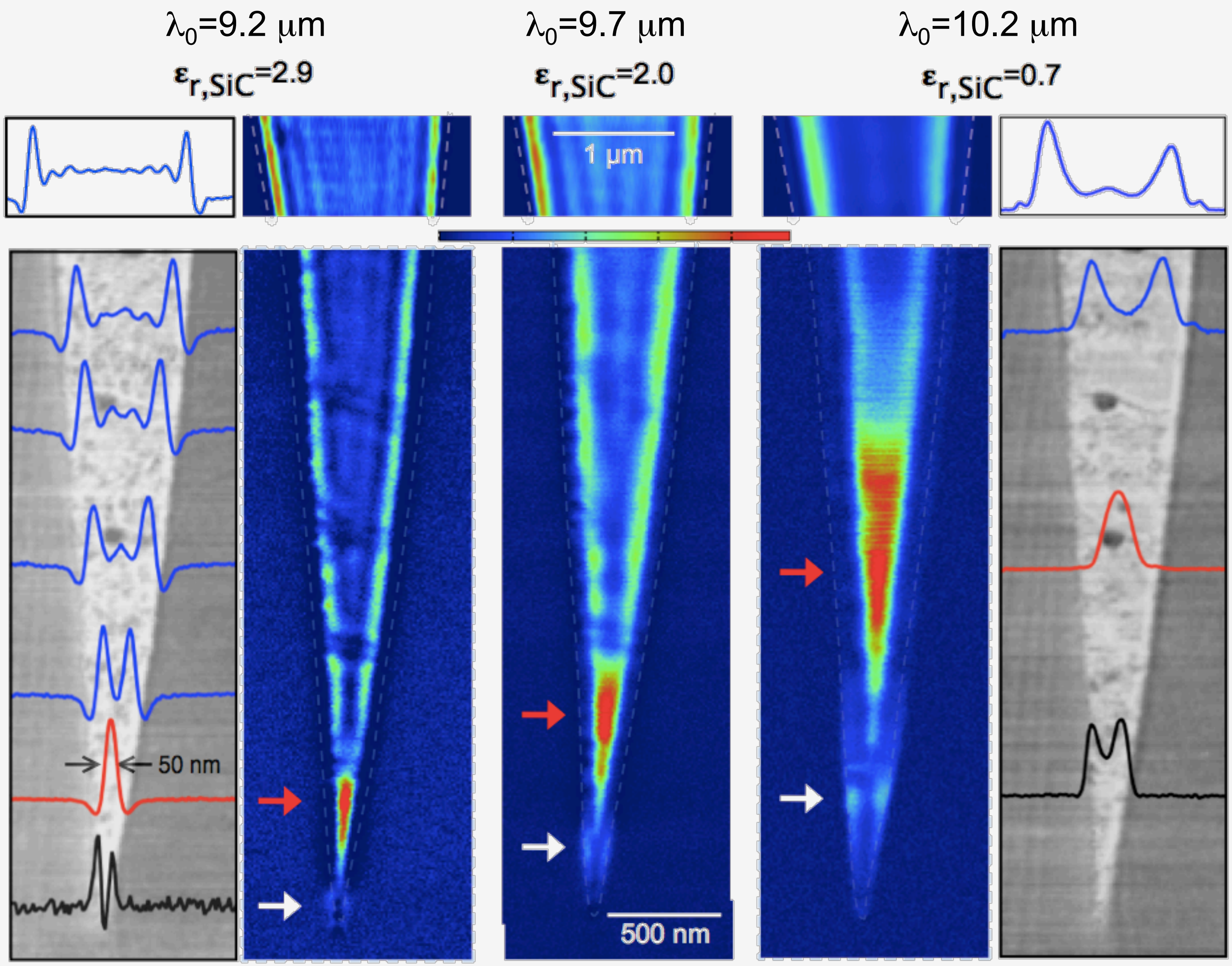
Graphene on SiC





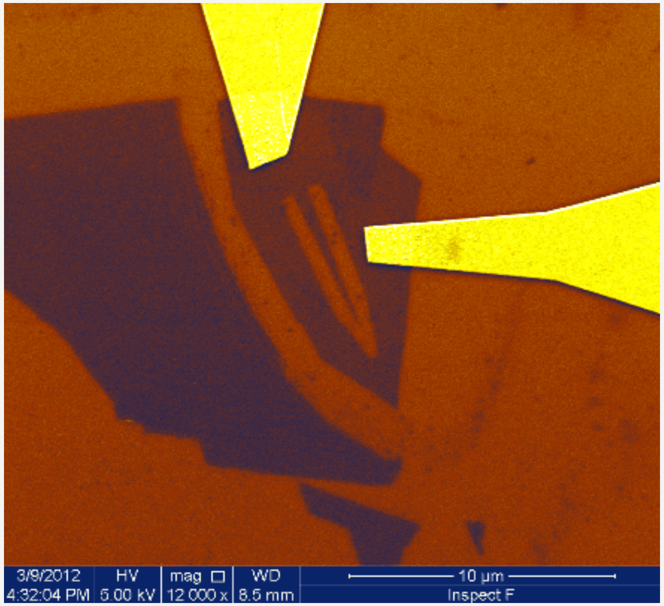
Camara *et al*., PRB (2009)

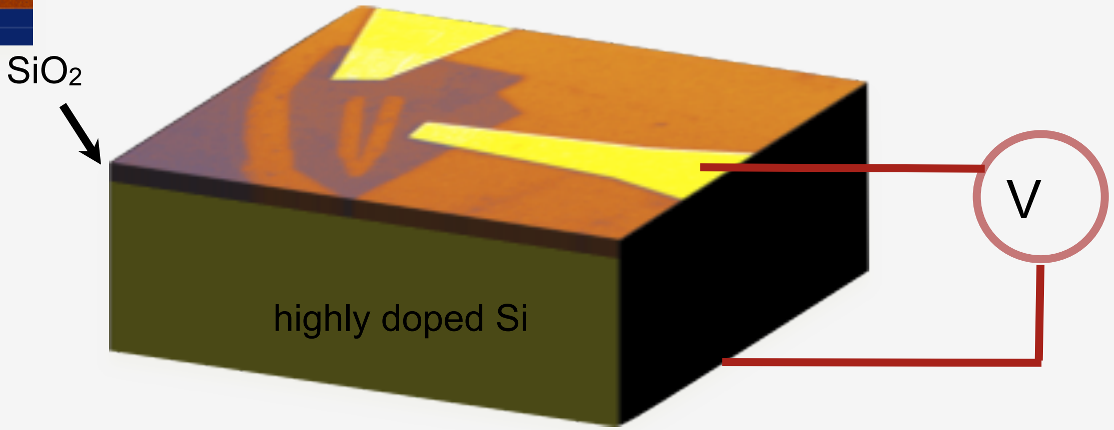
Experimental mapping of graphene plasmons



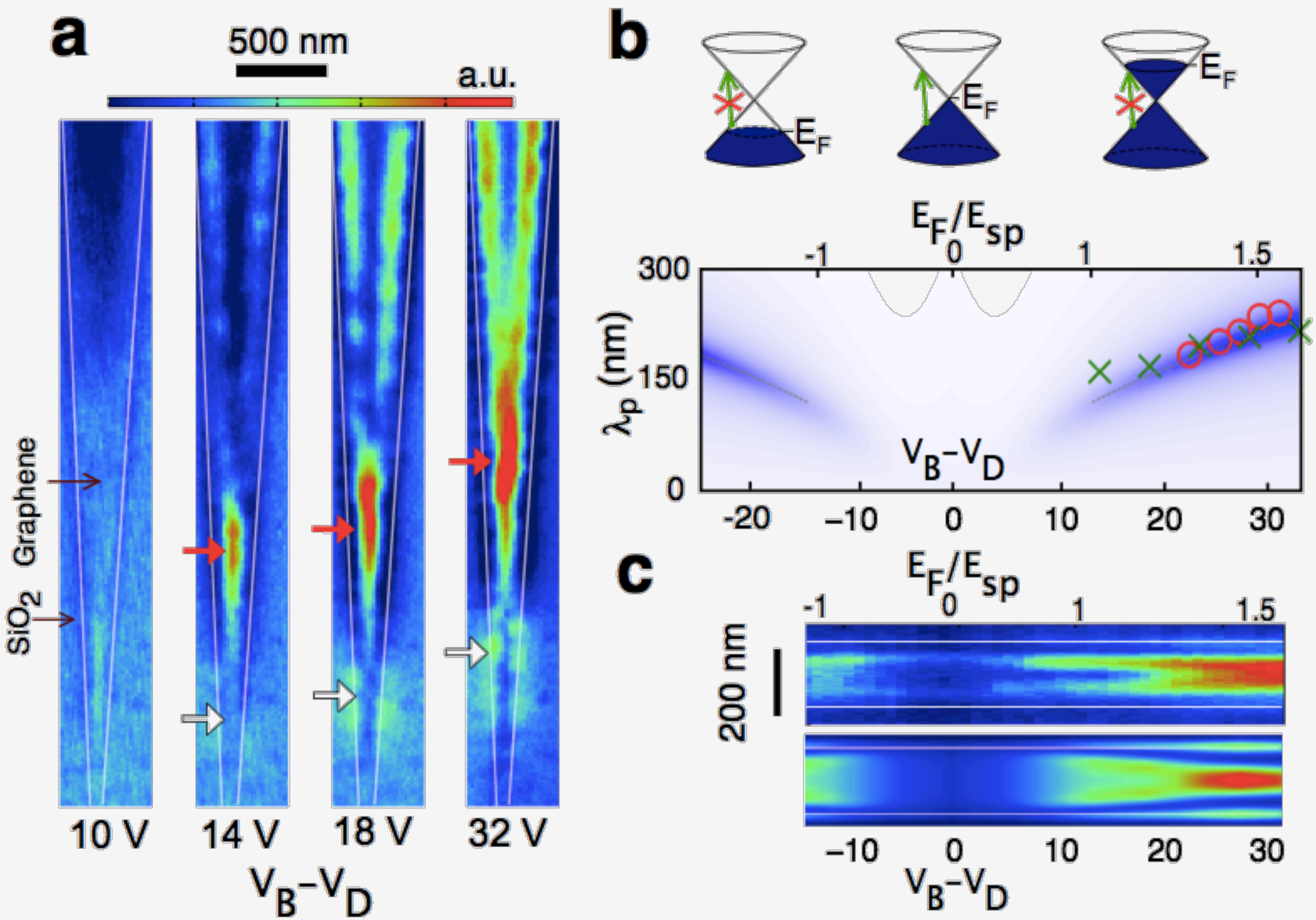
Chen *et al.*, Nature (2012)

Controlling optical fields by electric fields





Experimental mapping of graphene plasmons

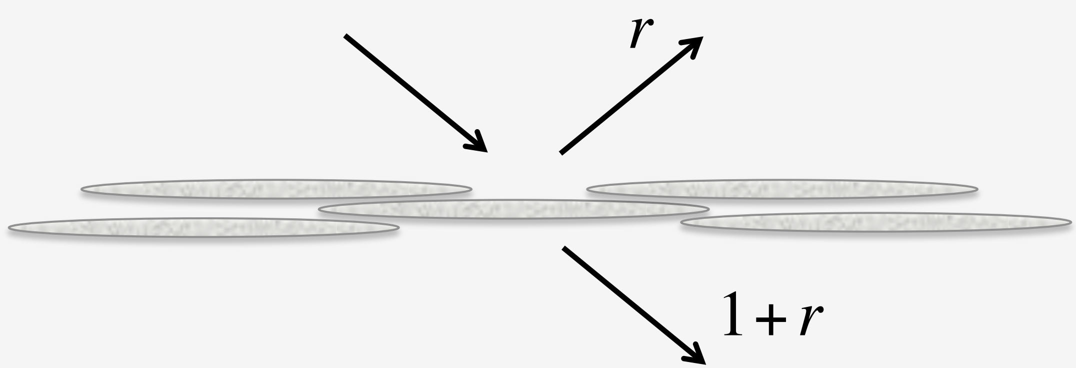


Chen *et al*., Nature (2012)

Complete optical absorption in graphene

Light absorption in graphene

$$\text{Absorption} \approx \pi/137 \approx 2.3\%$$



$$\text{Absorption} = 1 - |r|^2 - |1+r|^2 \quad \rightarrow \quad 50\% \text{ maximum}$$

Short historical overview of perfect absorption

nanophotonics.csic.es

Partially disordered silver films

- O. Hunderi and H. P. Myers, J. Phys. F: Metal Phys. 3, 683 (1973)

Diffraction in gratings, double-period metal gratings, and metamaterials

- M. C. Hutley and D. Maystre, Optics Communications 19, 431 (1976)
- D. Maystre and R. Petit, Optics Communications 17, 196 (1976)
- W.-C. Tan, J. R. Sambles, and T. W. Preist, Phys. Rev. B 61, 13177 (1999)
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- N. I. Landy et al., Phys. Rev. Lett. 100, 207402 (2008)
- N. Liu et al., Nano Lett. 10, 2342 (2010)

Doped silicon lamellar grating

- F. Marquier, M. Laroche, R. Carminati, J.-J. Greffet, Journal of Heat Transfer 129, 11 (2007)
- J.-J. Greffet, R. Carminati, K. Joulain, J.-P. Mulet, S. Mainguy, Y. Chen, Nature 416, 61 (2002)

Semiconductor and metal-semiconductor-metal nanostructures

- S. Collin, F. Pardo, R. Teissier, and J.-L. Pelouard, Appl. Phys. Lett. 85, 194 (2004)
- T.V. Teperik, F.J. García de Abajo, V.V. Popov, and M.S. Shur, Appl. Phys. Lett. 90 251910 (2007)

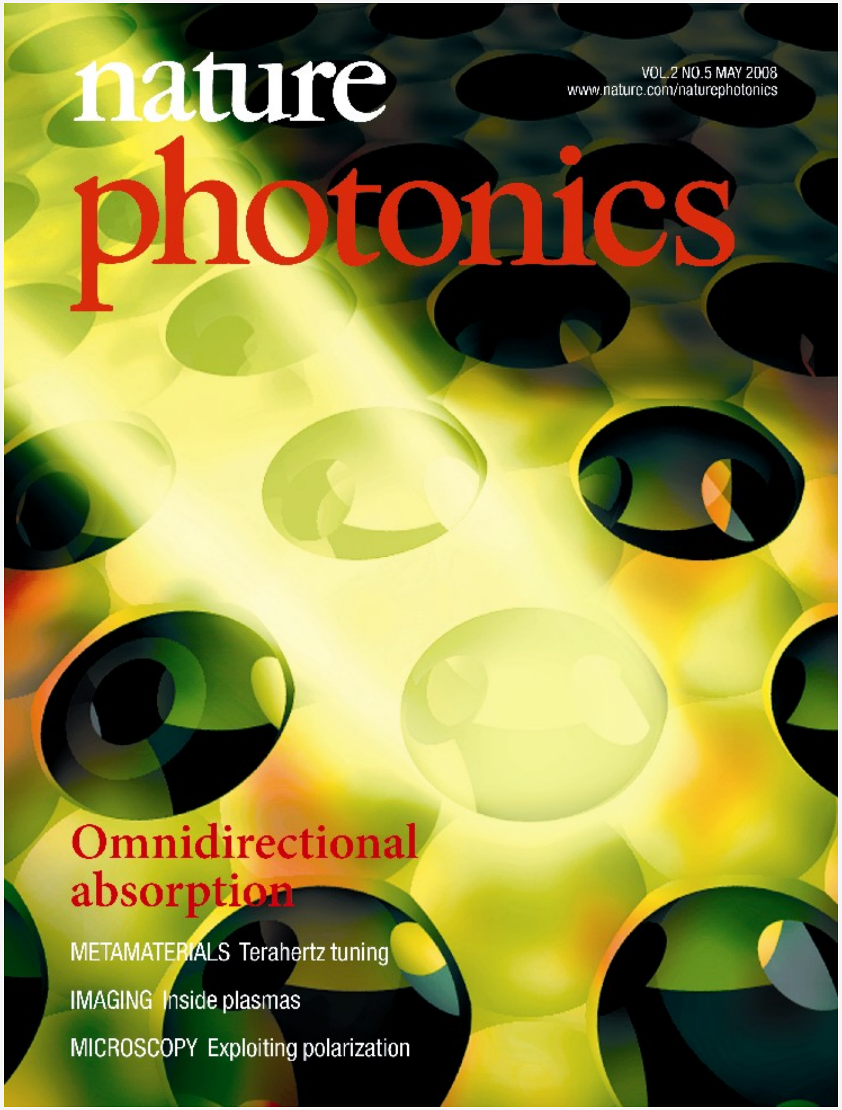
Multiplayer of metallic nanoparticles and nanopores in metal

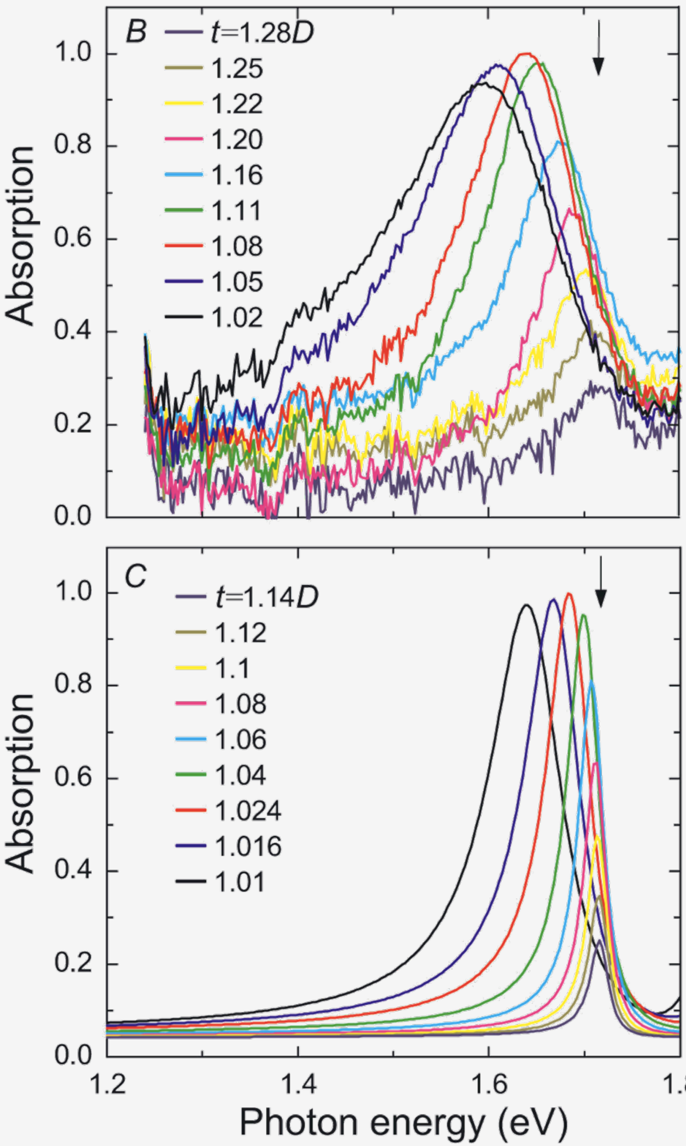
- T. V. Teperik, V. V. Popov, and F. J. García de Abajo, Phys. Rev. B 71, 085408 (2005)
- T. Teperik, V. Popov, and F. Garcıa de Abajo, J. Opt. A: Pure Appl. Opt. 0, 0 (2007)
- S.Kachan, O. Stenzel, and A. Ponyavina, Appl. Phys. B 84, 281 (2006)

Overdense plasma slab (in the microwave frequency range)

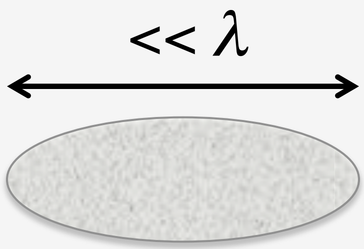
- Y. P. Bliokh, J. Felsteiner, and Y. Z. Slutsker, Phys. Rev. Lett. 95, 165003 (2005)

Nanovoids as perfect absorbers



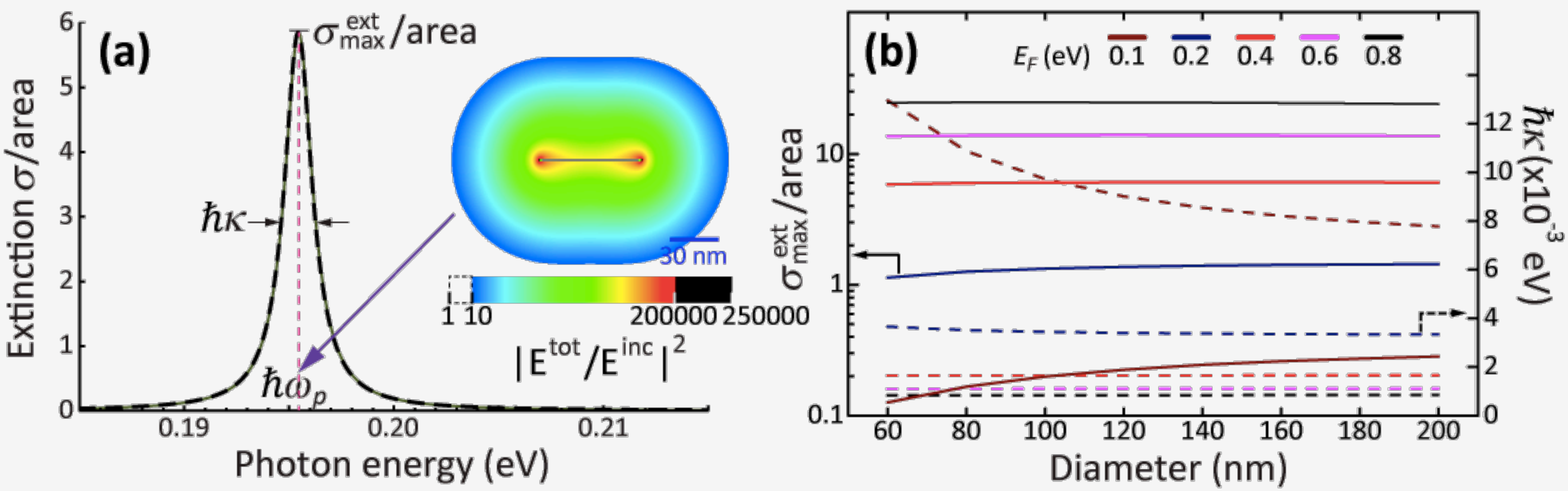


T.V.Teperik *et al.*, Nature Phot. (2008)



$$\alpha(\omega) = \frac{3c^3}{4\omega^3} \frac{\kappa_r}{\omega_p - \omega - i\kappa/2}$$

$$\sigma^{\mathrm{ext}}(\omega) = \frac{4\pi\omega}{c} \mathrm{Im}\{\alpha\} \approx \frac{3\lambda^2}{2\pi} \frac{\kappa_r}{\kappa}, \qquad \kappa_r << \kappa$$



Thongrattanasiri *et al.*, Phys. Rev. Lett. (2012)

Maximum absorption by a small particle

$$\alpha(\omega) = \frac{3c^3}{4\omega^3} \frac{\kappa_r}{\omega_p - \omega - i\kappa/2} \qquad \alpha(\omega_p) = \frac{3ic^3}{2\omega_p^3} \frac{\kappa_r}{\kappa}$$

$$\sigma^{\text{ext}}(\omega) = \frac{4\pi\omega}{c} \text{Im}\{\alpha\} \qquad \sigma^{\text{ext}}(\omega_p) = \frac{3\lambda^2}{2\pi} \frac{\kappa_r}{\kappa}$$

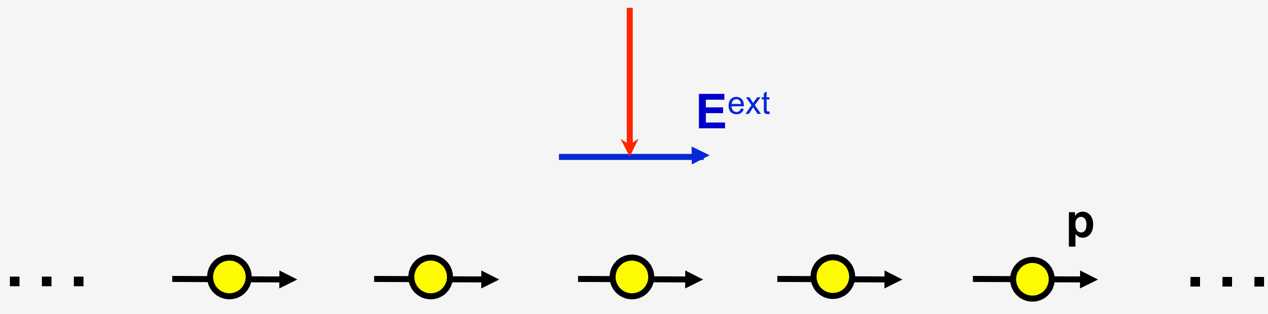
$$\sigma^{\text{abs}}(\omega) = \frac{4\pi\omega}{c} \left(\text{Im}\{\alpha\} - \frac{2\omega^3}{3c^3} |\alpha|^2 \right) \qquad \sigma^{\text{abs}}(\omega_p) = \frac{3\lambda^2}{2\pi} \frac{\kappa_r}{\kappa} \left(1 - \frac{\kappa_r}{\kappa} \right)$$

$$\sigma^{\text{ext}}_{\text{max}}(\omega_p) = \frac{3\lambda^2}{2\pi} \quad \text{for} \quad \kappa = \kappa_r$$

$$\sigma^{\text{abs}}_{\text{max}}(\omega_p) = \frac{3\lambda^2}{8\pi} \quad \text{for} \quad \kappa = 2\kappa_r$$

$$\boxed{\kappa_r = \kappa_a}$$

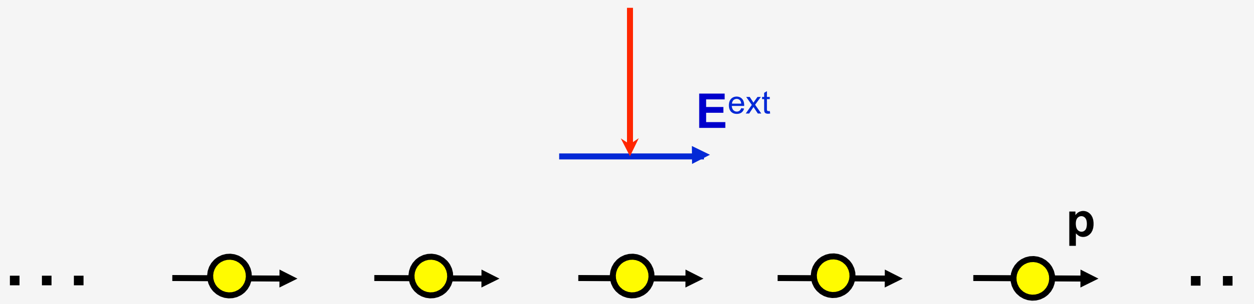
Description of graphene-disk array



$$\mathbf{p} = \alpha\left(\mathbf{E}^{ext} + \sum_{\text{lattice}} G_{\text{dip-dip}}\,\mathbf{p}\right),\qquad G_{\text{dip-dip}} \approx \frac{3\hat{\mathbf{r}}\hat{\mathbf{r}} - 1}{r^3}$$

$$\mathbf{p} = \frac{1}{\alpha^{-1} - G}\mathbf{E}^{ext}\qquad G = \sum_{\text{lattice}} G_{\text{dip-dip}}$$

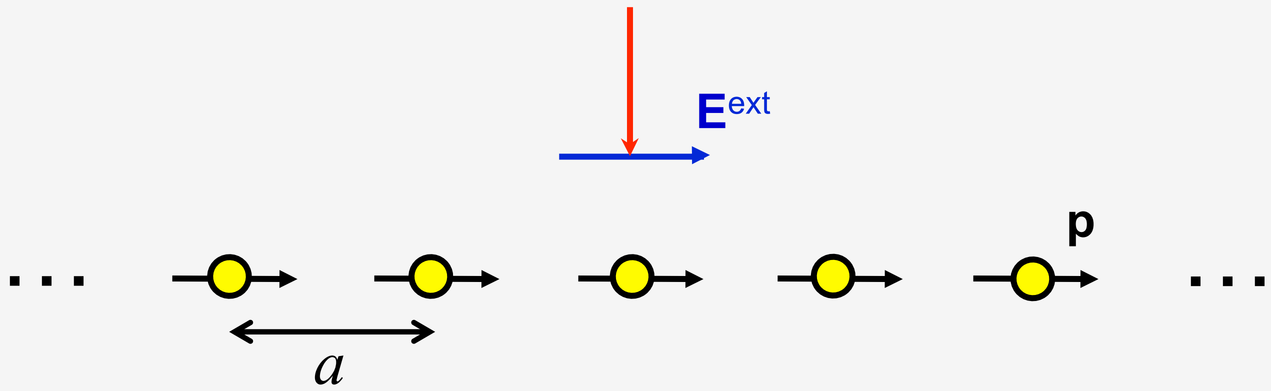
García de Abajo, Rev. Mod. Phys. (2007)



$$r = \frac{\pm iS}{\alpha^{-1} - G} \qquad S = \frac{2\pi\omega}{Ac}(\cos\theta)^{\mp 1}$$

$$\mathbf{p} = \frac{1}{\alpha^{-1} - G}\mathbf{E}^{ext} \qquad G = \sum_{\text{lattice}} G_{\text{dip-dip}}$$

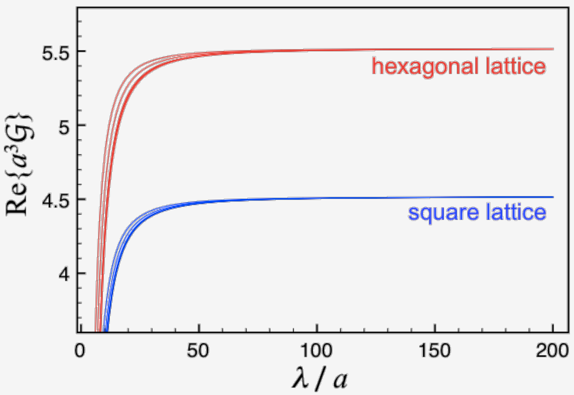
Description of graphene-disk array



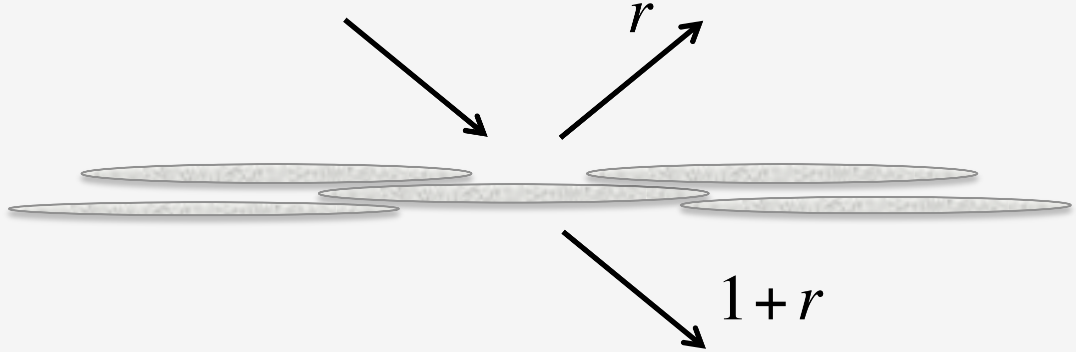
$$r = \frac{\pm iS}{\alpha^{-1} - G}$$

$$S = \frac{2\pi\omega}{Ac}(\cos\theta)^{\mp 1}$$

$$G \approx \frac{g}{a^3} + i\left(S - \frac{2\omega^3}{3c^3}\right)$$



Maximum absorption in graphene



$$\text{Absorption} = 1 - |r|^2 - |1+r|^2 \quad \rightarrow \quad 50\% \text{ maximum for } r = -1/2$$

$$r = \frac{\pm iS}{\alpha^{-1} - G} \qquad S = \frac{2\pi\omega}{Ac}(\cos\theta)^{\mp 1} \qquad G \approx \frac{g}{a^3} + i\left(S - \frac{2\omega^3}{3c^3}\right)$$

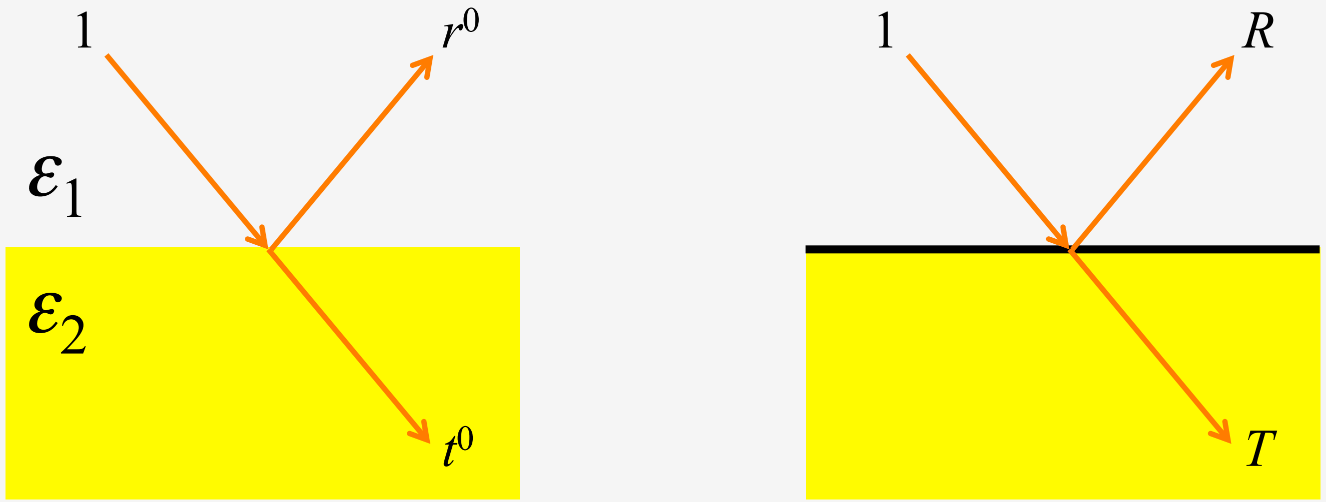
$$\alpha(\omega) = \frac{3c^3}{4\omega^3}\frac{\kappa_r}{\omega_p - \omega - i\kappa/2}$$

$$\boxed{\omega \approx \omega_p - 3g\kappa_r/4(\omega a/c)^3}$$

$$\boxed{\sigma^{\text{ext}}_{\text{max}} = 2A \times \begin{cases} \cos\theta, & s \text{ polarization}, \\ \cos^{-1}\theta, & p \text{ polarization}. \end{cases}}$$

Thongrattanasiri *et al.*, Phys. Rev. Lett. (2012)

Absorption in asymmetric environments

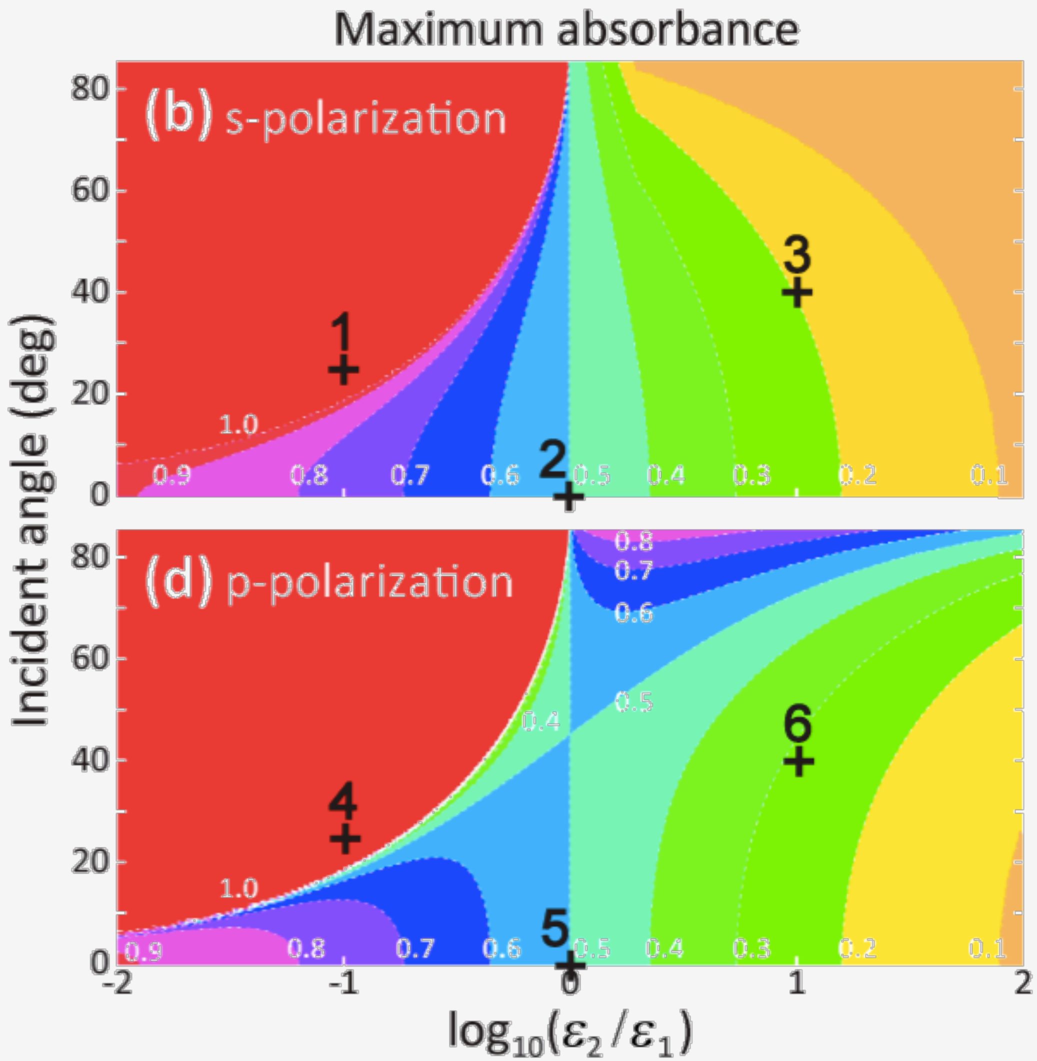


$$\eta = -\frac{r^0 \pm |r^0|^2 \pm \mathrm{Re}\{f\}|t^0|^2}{|1 \pm r^0|^2 + \mathrm{Re}\{f\}|t^0|^2}$$

$$\mathcal{A} = 1 - |r^0 + (1 \pm r^0)\eta|^2 - \mathrm{Re}\{f\}|t^0|^2|1 \pm \eta|^2$$

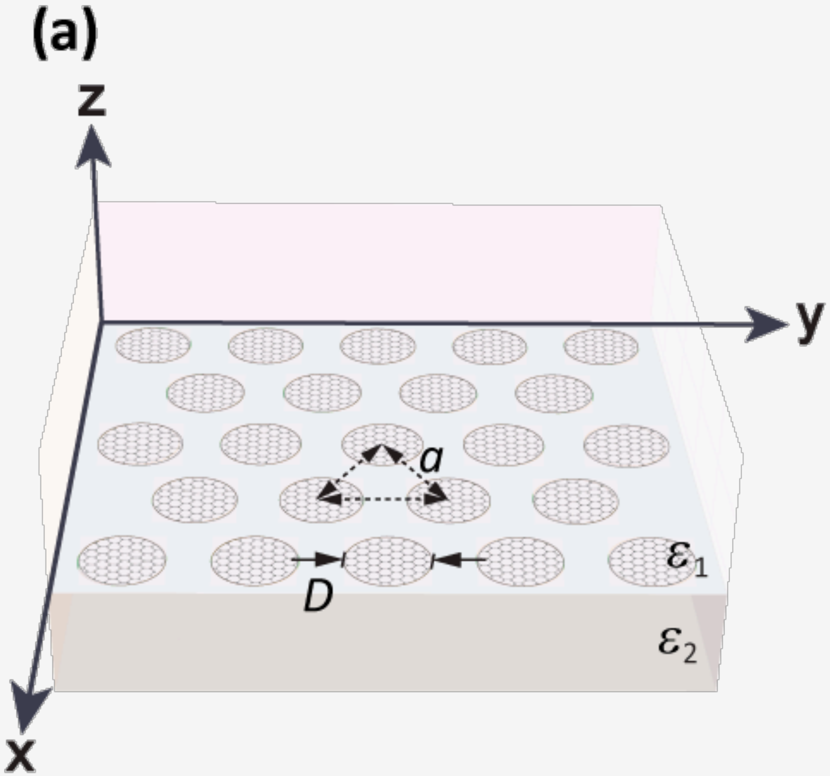
$$f = (\epsilon_2/\epsilon_1 - \sin^2\theta)^{1/2}/\cos\theta$$

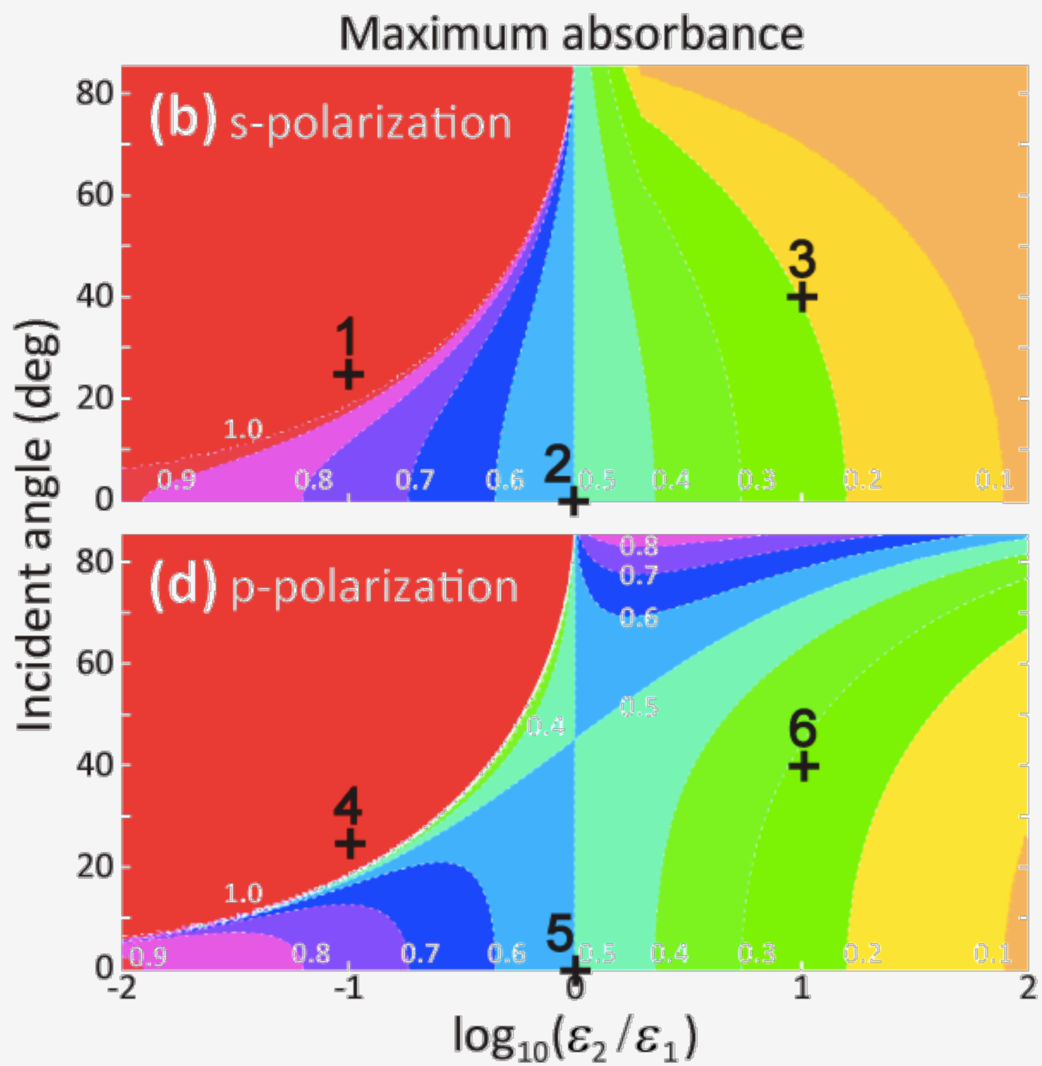
Absorption in asymmetric environments



Thongrattanasiri *et al.*, Phys. Rev. Lett. (2012)

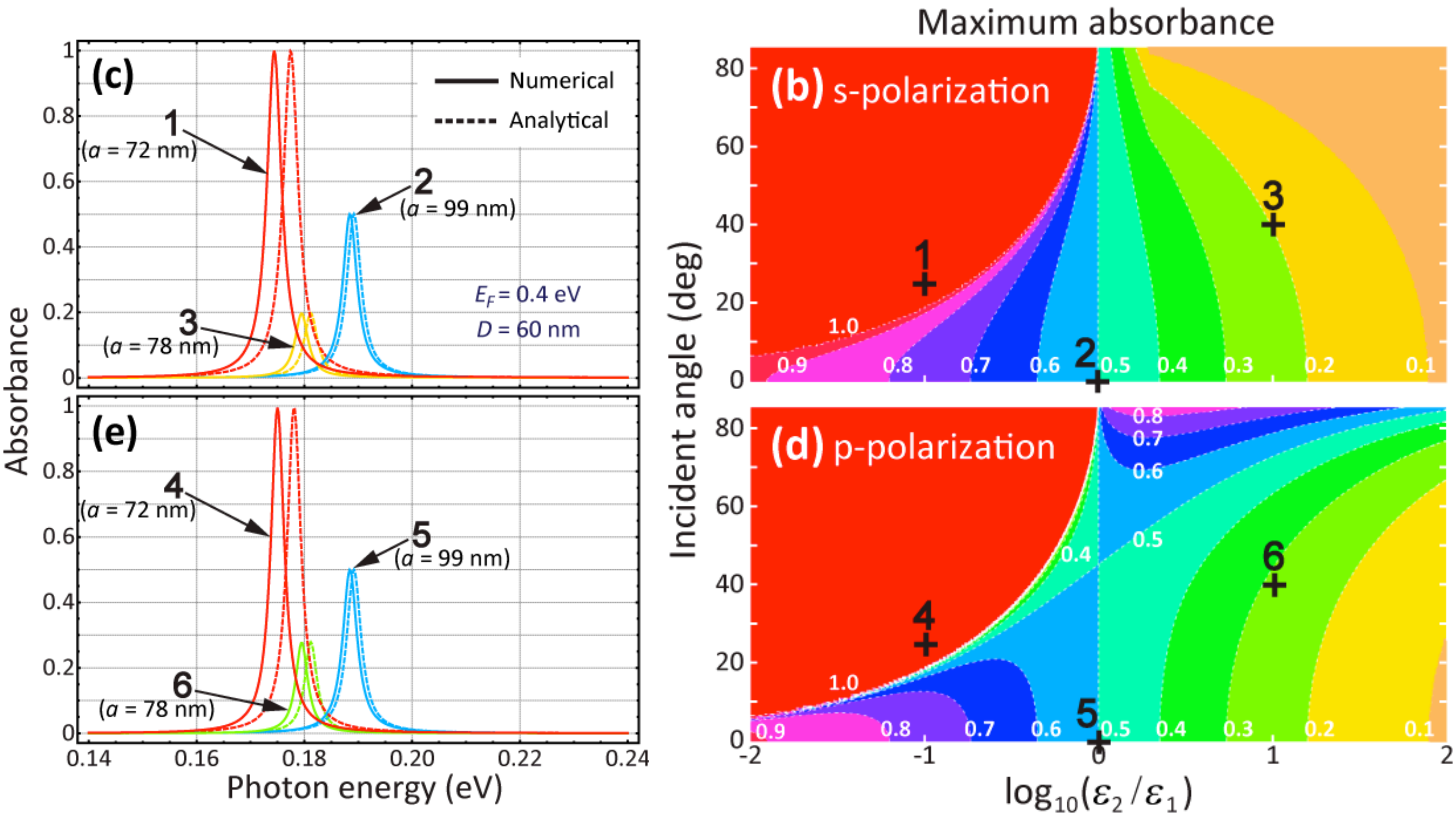
Absorption in asymmetric environments





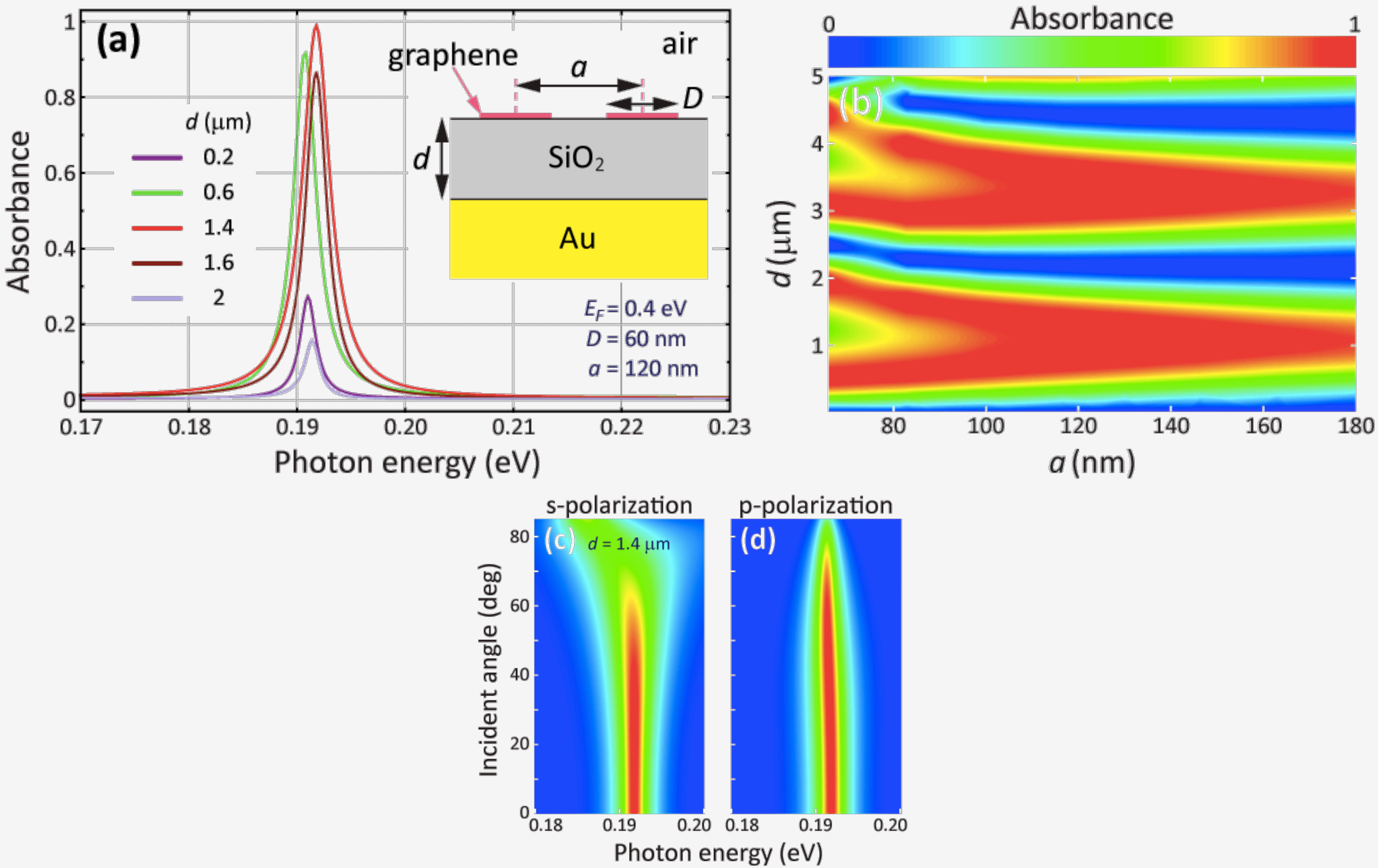
Thongrattanasiri *et al.*, Phys. Rev. Lett. (2012)

Absorption in asymmetric environments



Thongrattanasiri *et al*., Phys. Rev. Lett. (2012)

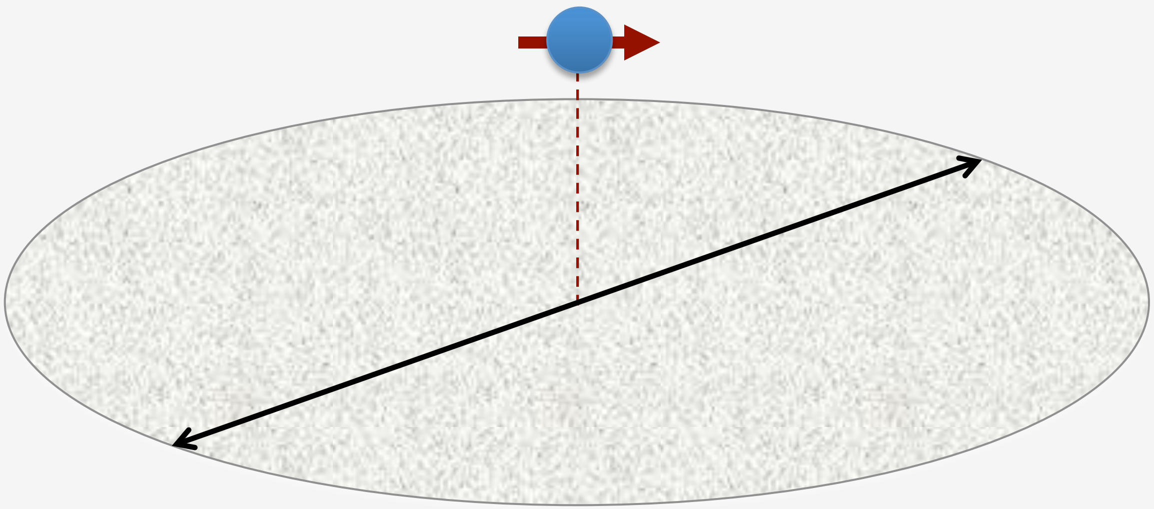
Perfect absorption



Thongrattanasiri *et al*., Phys. Rev. Lett. (2012)

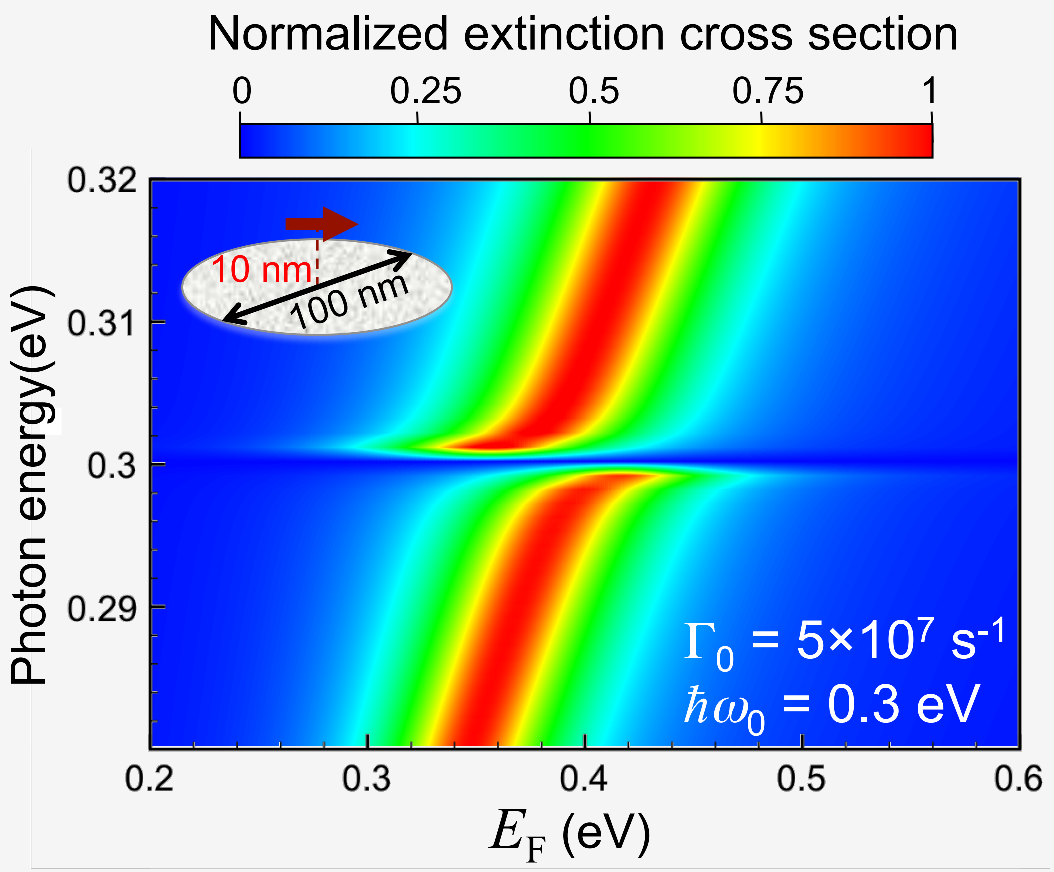
Strong light-matter interaction: quantum plasmonics with graphene

Vacuum Rabi splitting



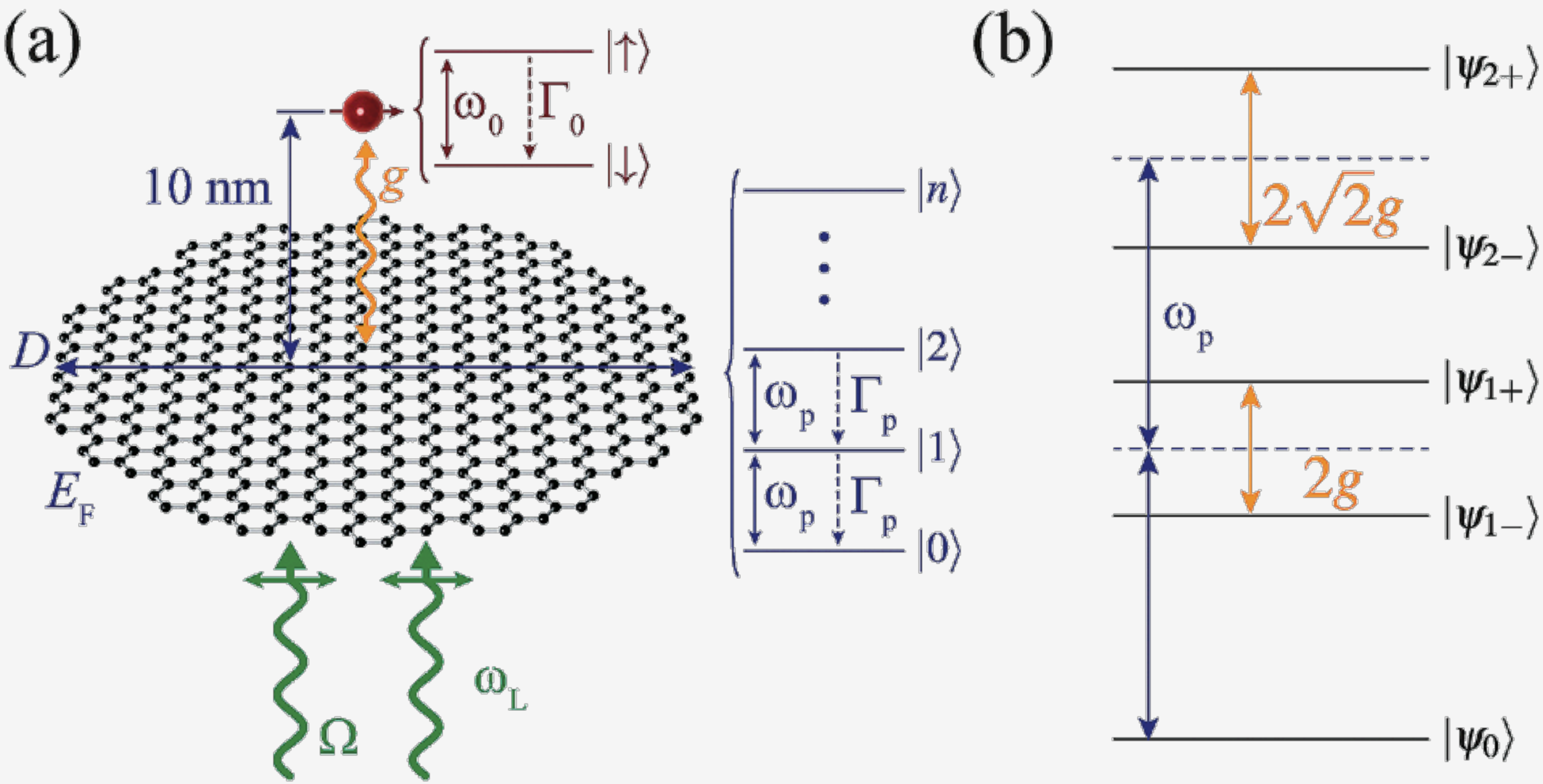
Koppens, Chang, and García de Abajo, Nano Lett. (2011)

Vacuum Rabi splitting



Koppens, Chang, and García de Abajo, Nano Lett. (2011)

Jaynes-Cummings ladder

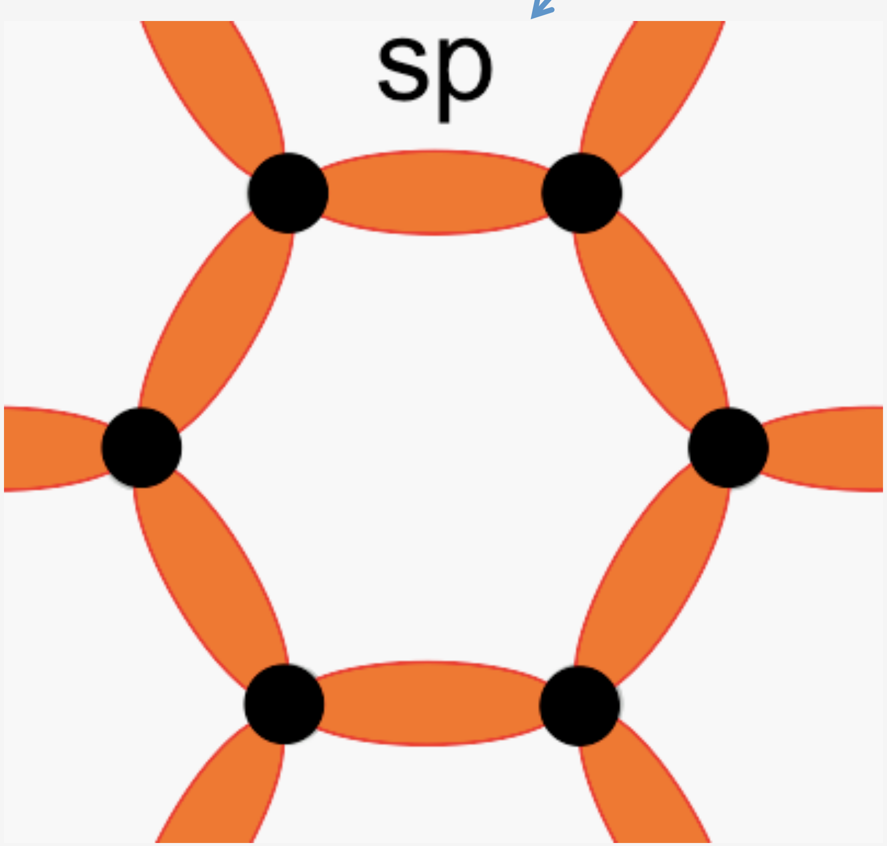


Manjavacas *et al.*, ACS Nano (2012)

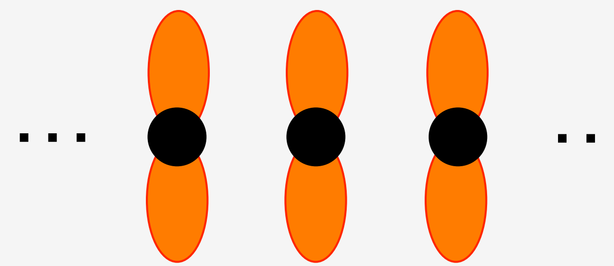
Quantum effects in graphene plasmons

Quantum effects in graphene plasmons

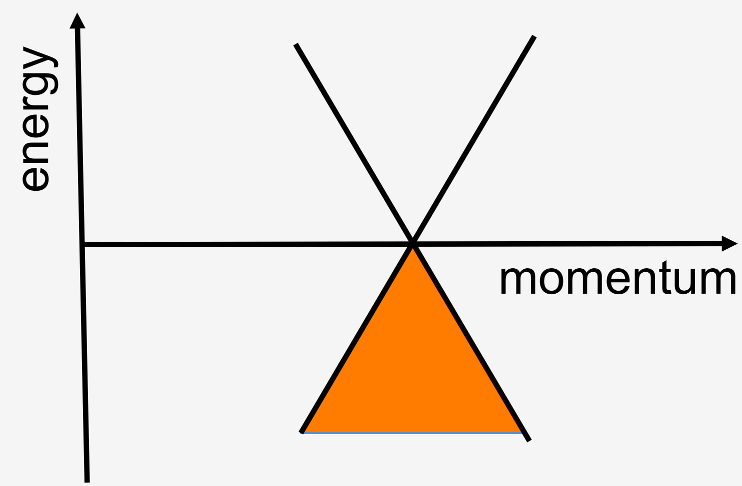
$1s^2\,2s^2\,2p^2 \rightarrow 1s^2\,(2sp^2)^3\,2p^1$



$2p_z$

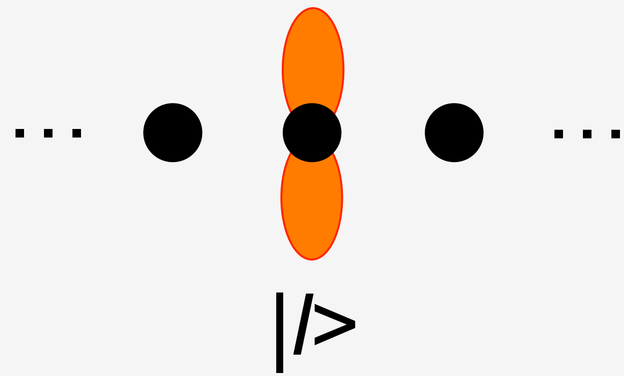


$|l\rangle$



Electron state j
of energy ε_j

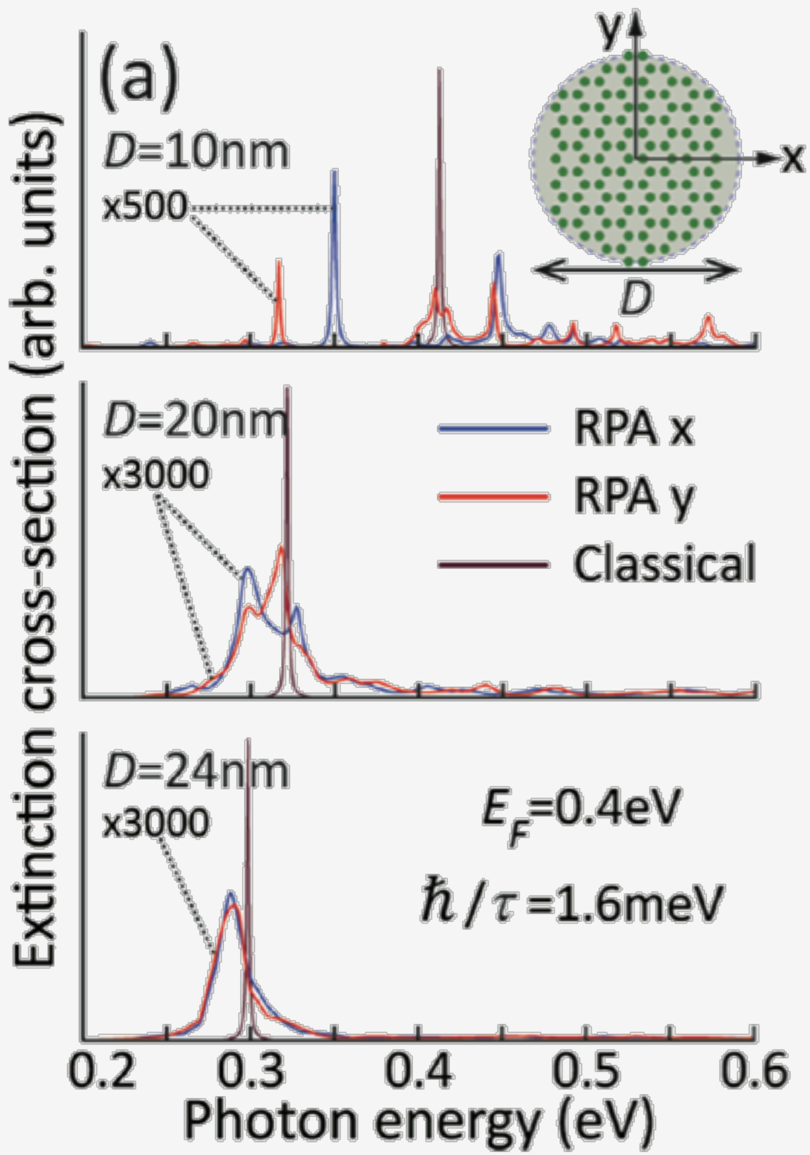
$\sum_l a_{jl}|l\rangle$



RPA response

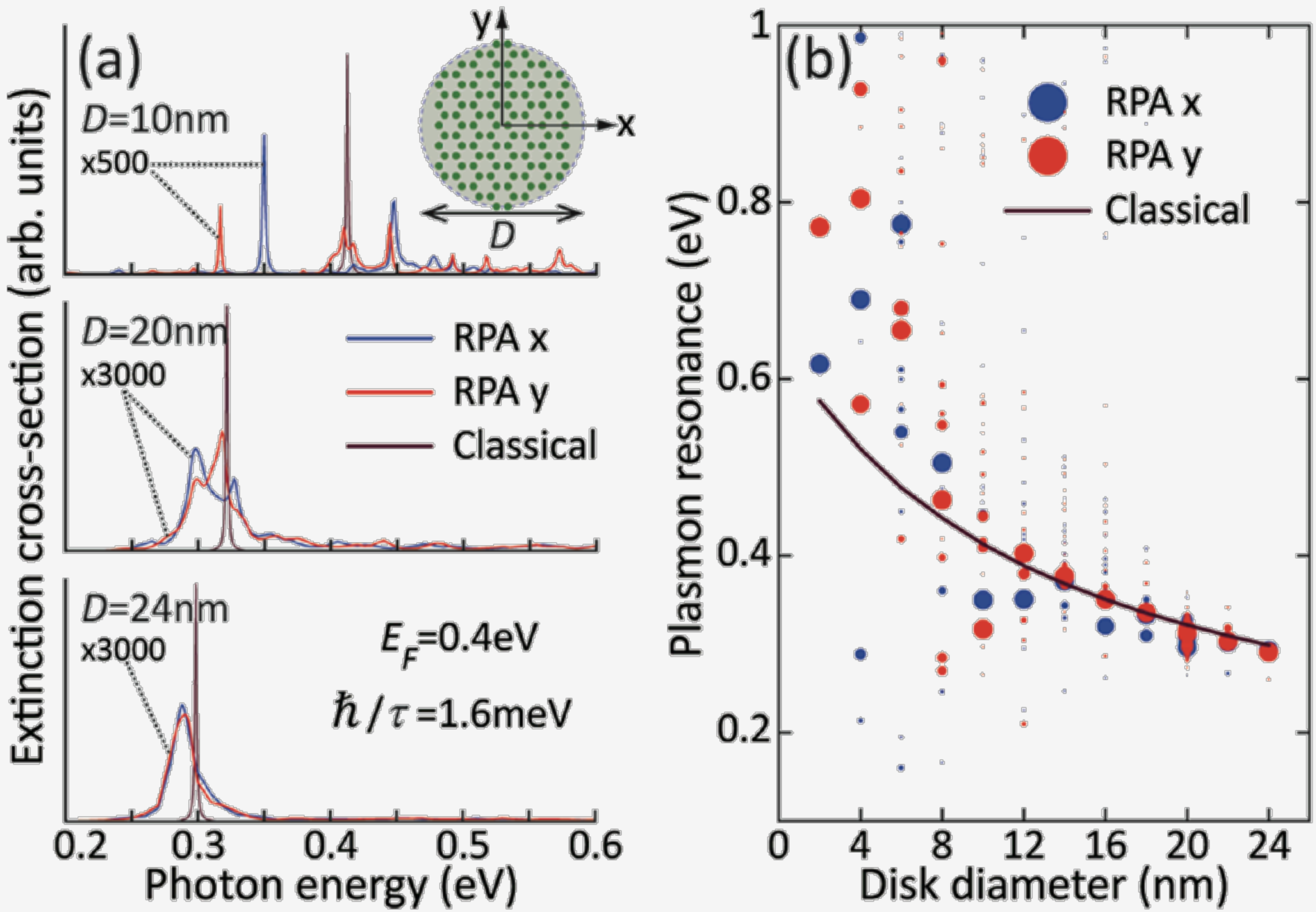
$$\chi^0_{ll'}(\omega) = \frac{2e^2}{\hbar}\sum_{jj'}(f_{j'} - f_j)\frac{a_{jl}a^*_{jl'}a^*_{j'l}a_{j'l'}}{\omega - (\varepsilon_j - \varepsilon_{j'}) + i/2\tau}$$

Thongrattanasiri *et al.*, ACS Nano (2012)



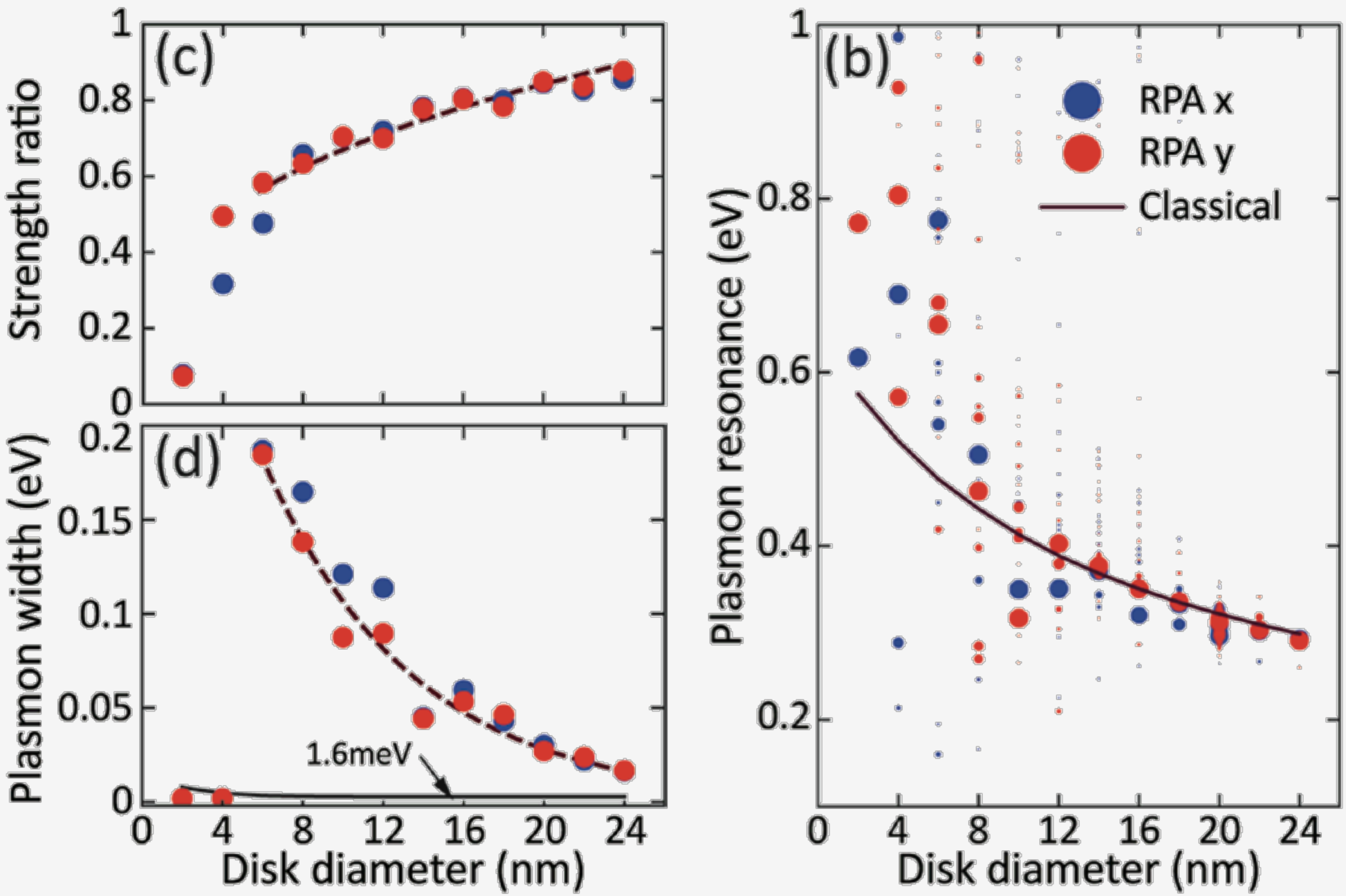
Thongrattanasiri *et al*., ACS Nano (2012)

Quantum effects in graphene plasmons



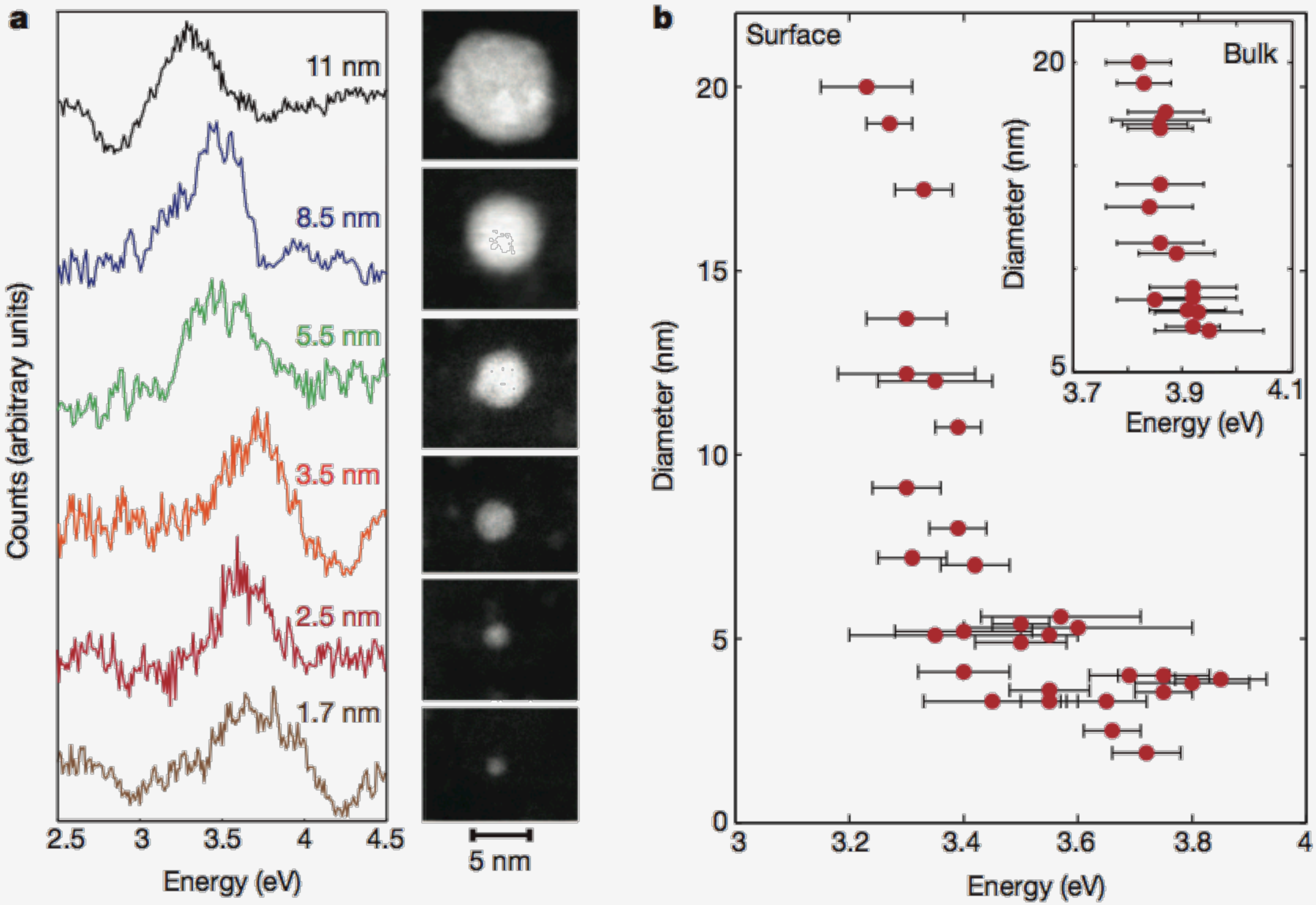
Thongrattanasiri *et al.*, ACS Nano (2012)

Quantum effects in graphene plasmons



Thongrattanasiri *et al.*, ACS Nano (2012)

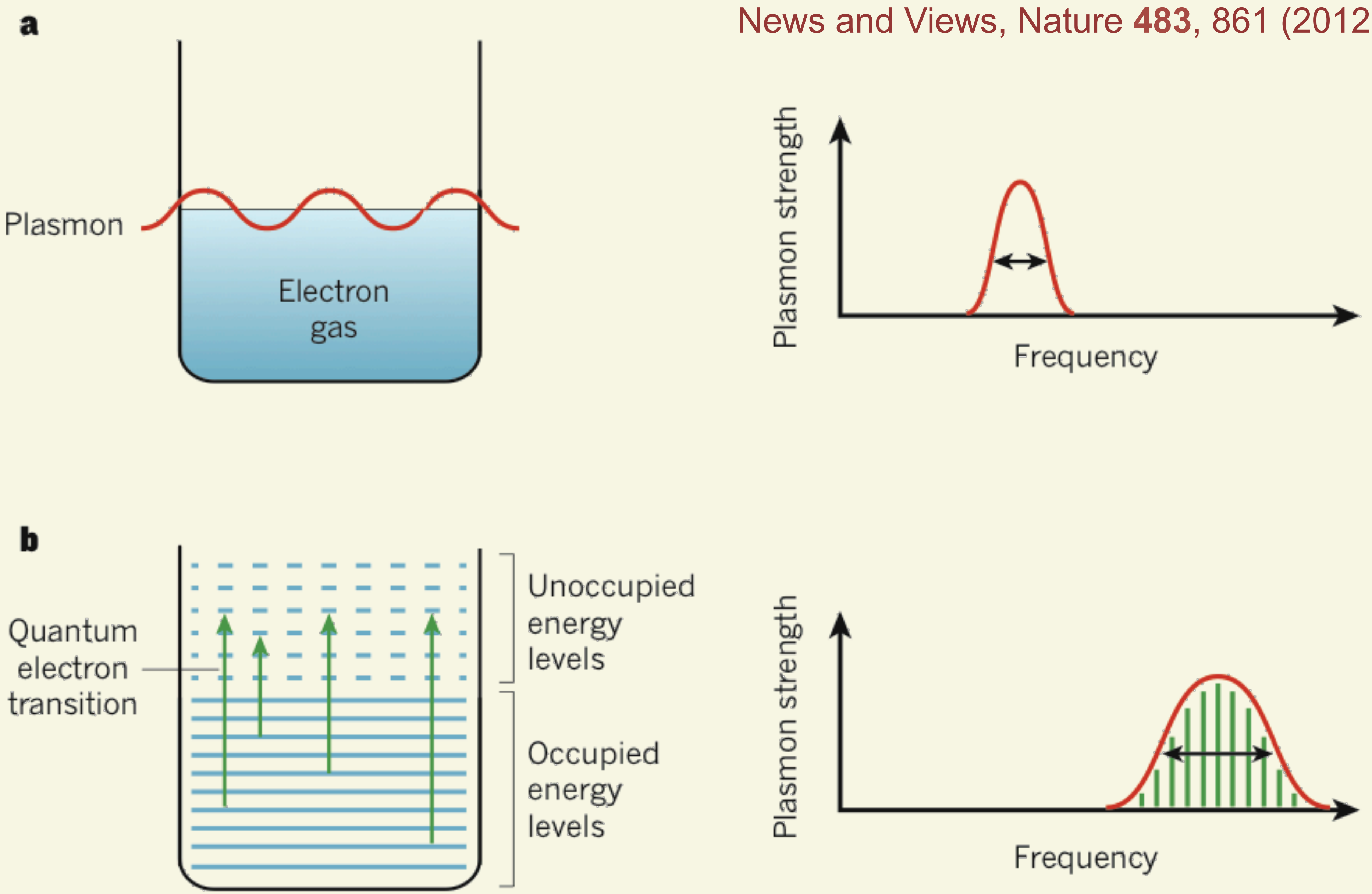
Quantum effects in silver plasmons



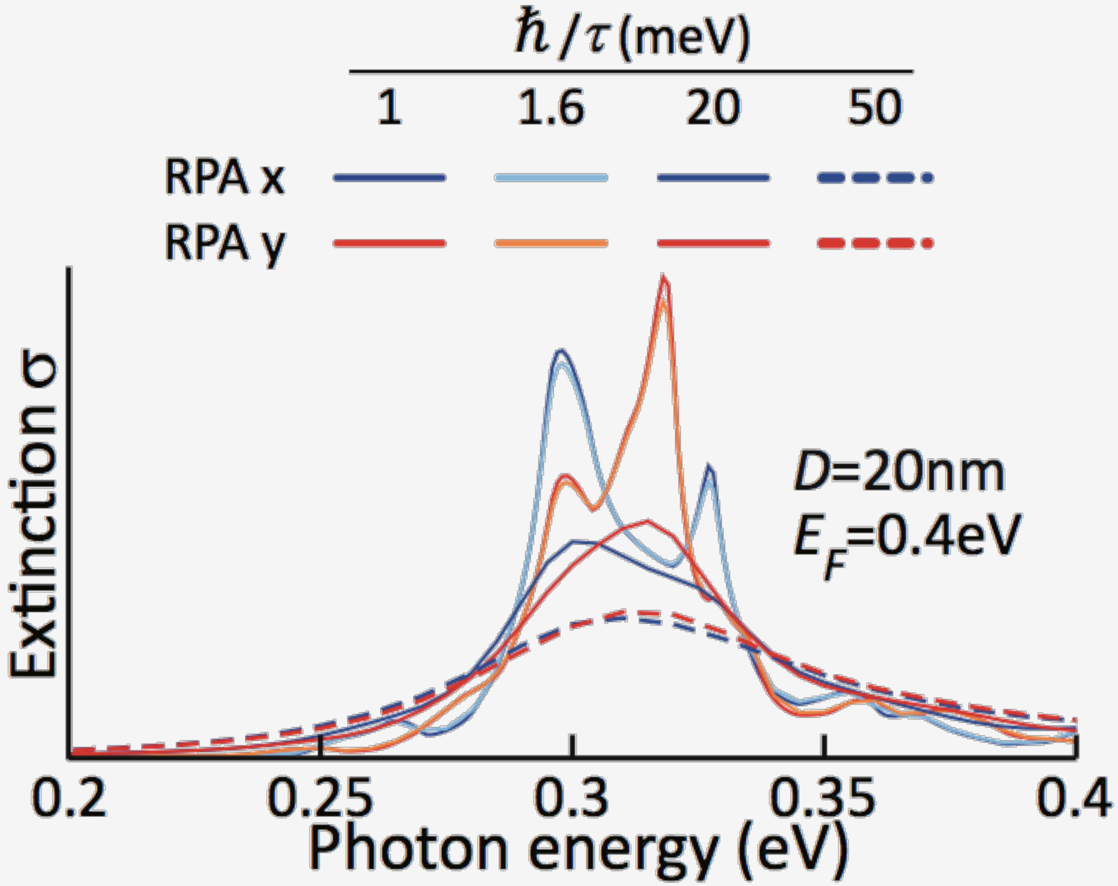
Scholl, Koh, and Dionne, Nature (2012)

Quantum effects in graphene plasmons

News and Views, Nature **483**, 861 (2012)



Quantum effects in graphene plasmons

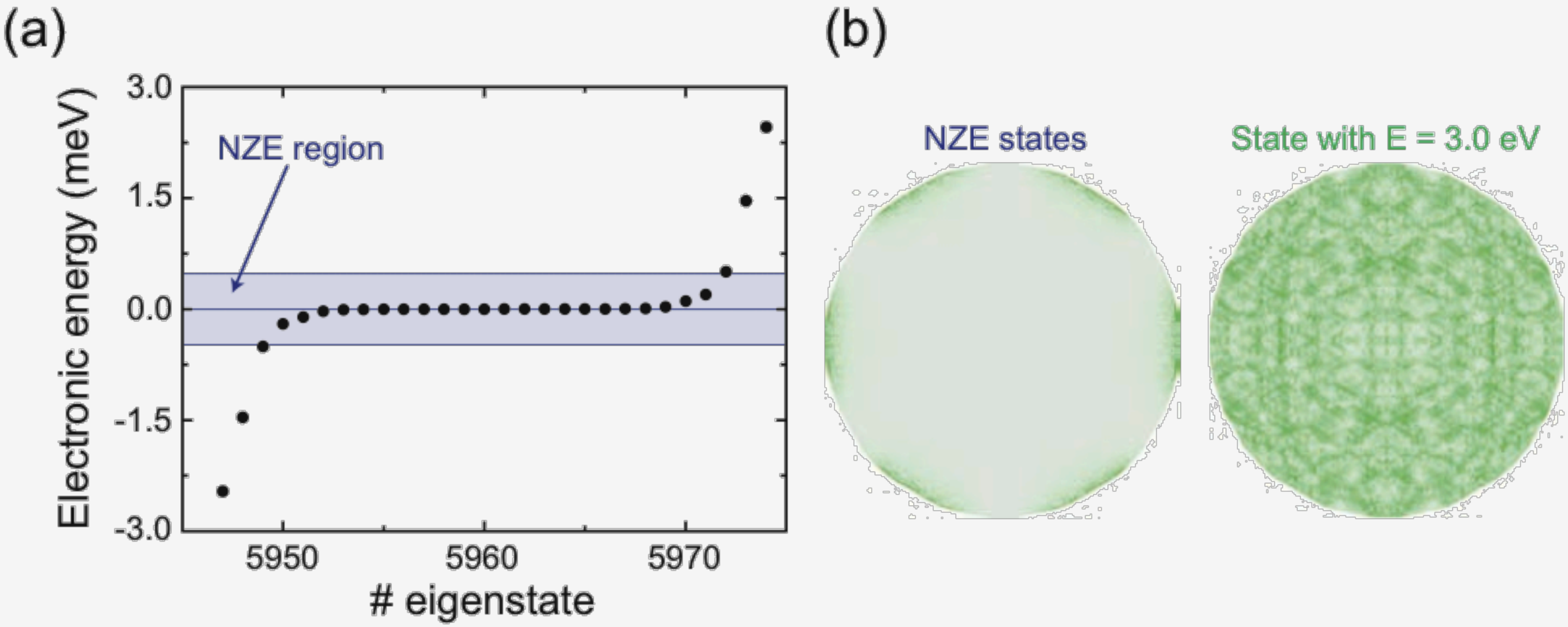


$$\chi^0_{ll'}(\omega) = \frac{2e^2}{\hbar}\sum_{jj'}(f_{j'} - f_j)\frac{a_{jl}a^*_{jl'}a^*_{j'l}a_{j'l'}}{\omega - (\varepsilon_j - \varepsilon_{j'}) + i/2\tau}$$

Thongrattanasiri *et al.*, ACS Nano (2012)

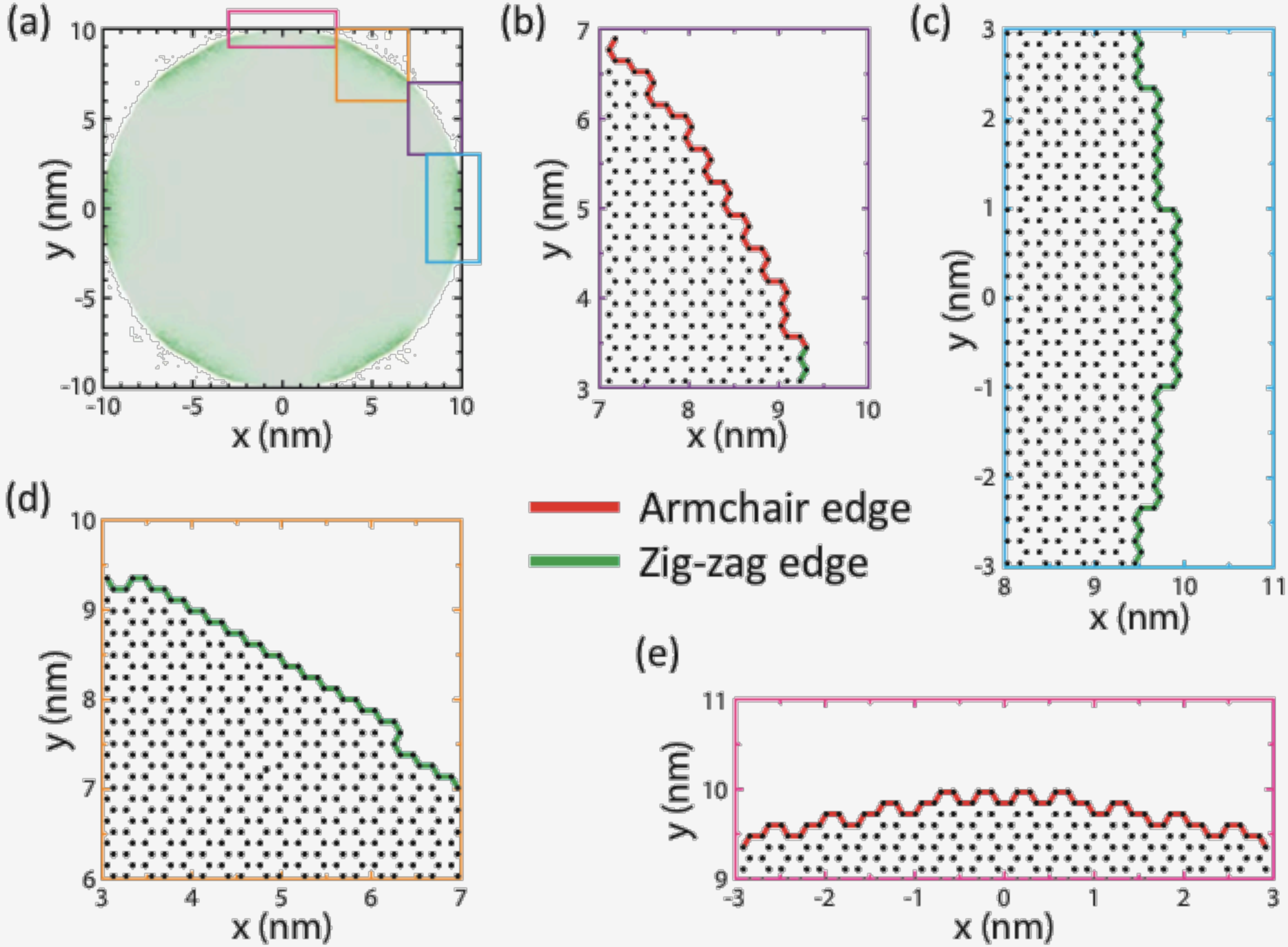
Electron state *j*
of energy ε_j

$$\sum_l a_{jl} |l\rangle$$



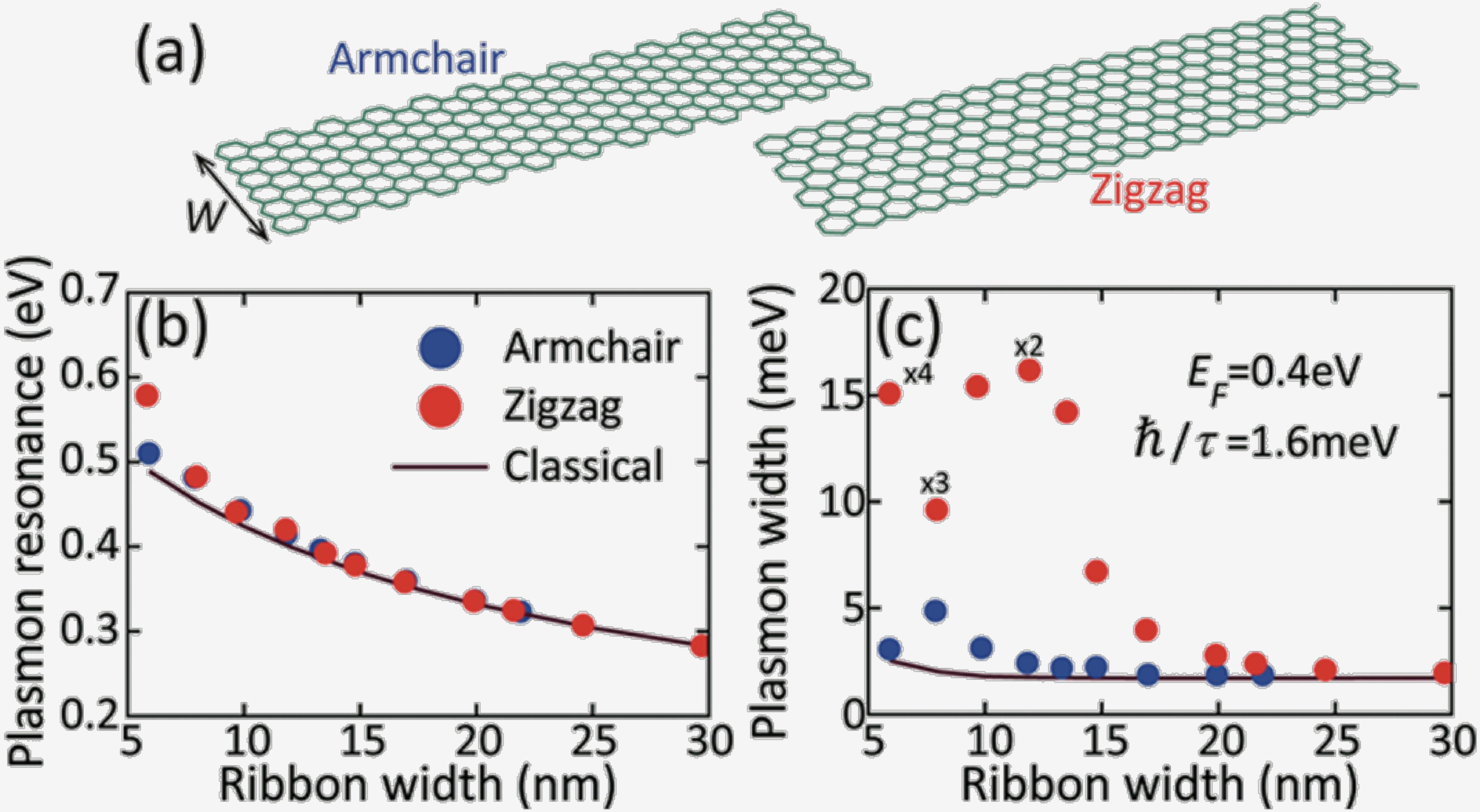
Thongrattanasiri *et al.*, ACS Nano (2012)

Quantum effects in graphene plasmons



Thongrattanasiri *et al.*, ACS Nano (2012)

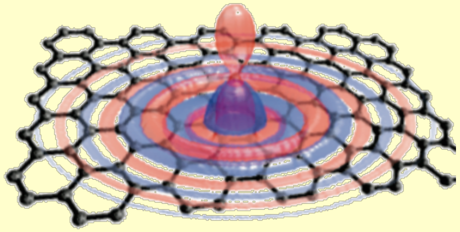
Quantum effects in graphene plasmons



Thongrattanasiri *et al.*, ACS Nano (2012)

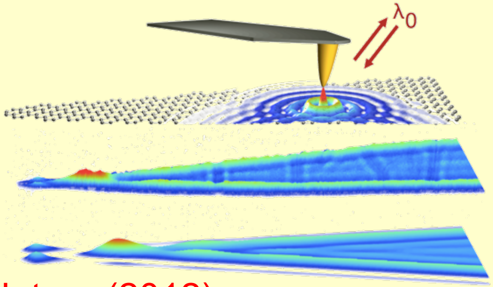
Graphene plasmonics

Strong light-matter interaction



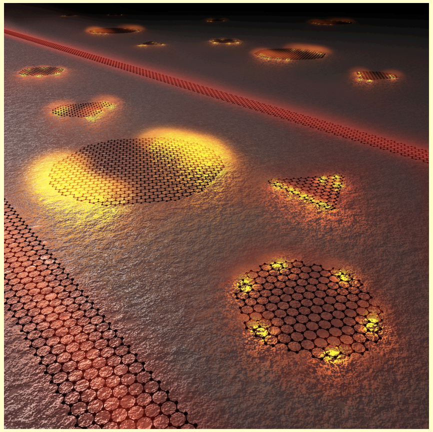
Koppens, Chang & García de Abajo, Nano Lett. (2011)

Experimental observations



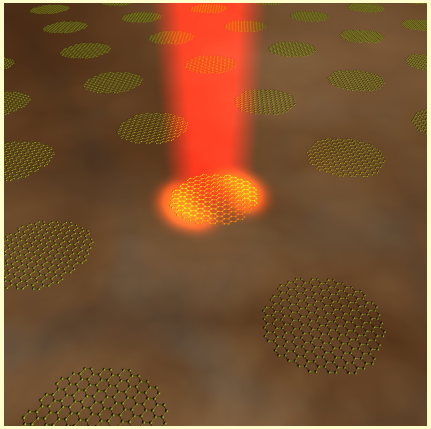
Basov's group, Nature (2012)
Koppens, Hillenbrand, García de Abajo's gropus, Nature (2012)
Zheyu et al, in preparation

Intrinsic quantum effects



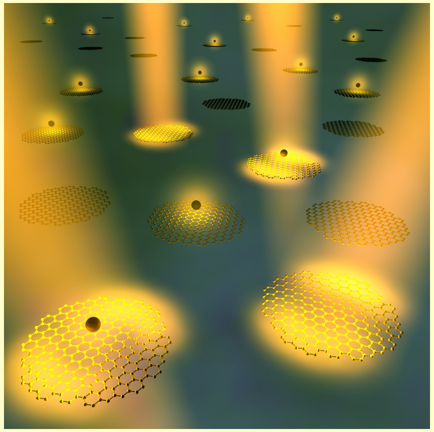
Thongrattanasiri, Manjavacas & García de Abajo, ACS NANO (2011)

Extraordinary metamaterials: Complete optical absorption



Thongrattanasiri, Koppens & García de Abajo, PRL (2011)

Quantum optics with graphene plasmons



Manjavacas, Nordlander & García de Abajo, ACS NANO (2011)